ERRATA

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43	11	eclipsed area	geographical coordinates system
44	7 (from the bottom)	$P^{M}_{N}=P^{M}_{qN}-P^{M}_{eN}$	$P^{M}_{N}=P^{M}_{qN}-P^{M}_{qN}$

A Note of the Effect on the Geomagnetic Field of the Solar Eclipse

By

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概　要

地磁気静穏日，日変化に及ぼす日食の影響は，従来一様な日変化磁場を仮定して評価されてきたが，この近似は稍々粗雑すぎるので，通常の日変化磁場を採用してより一般的に影響の量を評価する方法を述べた．

§ 1. Introduction

The effect of the solar eclipse on the geomagnetic field has been studied by many authors, from observational or theoretical stand point. S. Chapman stated the idea that the solar eclipse diminishes the conductivity in the ionosphere and the current systems of the Sq field are deformed. He calculated, regarding an theoretical model and estimated the effect is at most 10 or 20 γ. 〔1〕

From the same view point, T. Nagata et al and H. Volland studied the effects theoretically. 〔2〕, 〔3〕 In these studies, they assumed the uniform Sq current system. However, the diameter of the penumbral eclipse region is about 4000 miles, and tho assumption that the current system of Sq field are uniform over such extensive area, may be unreasonable. 〔1〕 Especially, in case of the solar eclipse 19 th, April 1958 the center of the annular eclipse zone goes on by the side of the vortex center of the Sq current system. In this case, the assumption will be rather rough. In this paper, the author wishes to exclude the assumption by proceeding along the dynamo theory. Furthermore, H. Volland who treated the problem as the dynamo theory of the Sq, supposes that the current functions can be superposed, that is, $R=Re+Rq$, where Rq is the current function for the conductivity which produces the Sq field and Re for the deforming conductivity, on the eclipsed day. In case of the two dimensional problem, that is generally unreasonable. The points are treated more generally in this paper.

§ 2. Determination of the current function

The electric current in the E or lower produces the Sq field and the dynamo theory explains the procedure. The effects of the solar eclipse on the Sq field have been also explained from the view point that the deformation of the Sq current system is arisen on account of diminishing of the conductivity in the same layer.

Taking the polar coordinates system, the dynamo equation in the two dimension can be written as follows;

$$a\sigma^2\left[\frac{\partial(vH_z)}{\partial\phi}+\frac{\partial(uH_z\sin\theta)}{\partial\theta}\right]=\sigma\left[\frac{1}{\sin\theta}\frac{\partial^2 R}{\partial\phi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial R}{\partial\theta}\right)\right]-\left[\frac{1}{\sin\theta}\frac{\partial R}{\partial\phi}\frac{\partial\sigma}{\partial\phi}+\sin\theta\frac{\partial R}{\partial\theta}\frac{\partial\sigma}{\partial\theta}\right] \quad (1)$$

where H_z is the vertical component of the earth's magnetic field and u, v the southward and eastward components of the motion of the medium. R is the current function.

If the earth's magnetic field is regarded as that of a centered magnetic dipole, then we have approximately

$$H_z=C\cos\theta+C\tan\theta_0\sin\theta\cos(\phi-\phi_0) \quad (2)$$

where C is a constant and is approximately equal to -2/3 gauss, θ_0 and ϕ_0 the coordinates of the geomagnetic north pole.

If we assume that the oscillation of the ionospheric layer, where the dynamo current flows, is of the harmonic type and has a velocity potential Ψ given by

$$\Psi=\sum_m\sum_n K_n^m P_n^m \sin[mt-\alpha_n^m] \quad (3)$$

then $$u=-\frac{\partial\Psi}{a\partial\theta} \quad \text{and} \quad v=-\frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\phi}$$

The nature of the oscillation is not known accurately, but, whatever be the actual form, it is possible to express it by the above.

Following Chaman, we assume that the integrated conductivity in a spherical shell where the dynamo current is produced, is given by

$$\sigma_q=a_0+a_1\cos\omega+a_2\cos^2\omega \quad (4)$$

where $$\cos\omega=\sin\delta\cos\theta+\cos\delta\sin\theta\cos t$$

δ is the declination of the sun, θ and λ are the colatitude and longitude of the point, and t the local time.

The conductivity σ in the same layer on the eclipsed day are different from σ_q and the difference between them is the effect of the solar eclipse on the conductivity. When we define,

$$\sigma_e=\sigma_q-\sigma,\quad \mathcal{E}=\frac{\sigma_q-\sigma_{min}}{\sigma_q},\quad \Gamma=\frac{\sigma_q-\sigma}{\sigma_q-\sigma_{min}} \quad (5)$$

the following can be obtained.

$$\sigma=\sigma_q(1-\mathcal{E}\cdot\Gamma) \quad (6)$$

σ_{min} is the minimum conductivity in the eclipsed area and $\mathcal{E}$ depends on the recombination coefficient and can be regarded as constant during the solar eclipse. $\mathcal{E}$ and Γ are determined from the observation of the ionosphere.

For instance we assume, following Volland, 〔3〕

$$\Gamma(\Theta)=\frac{1}{128}(34+39\cos 2\Theta+30\cos 4\Theta+25\cos 6\Theta) \qquad \Theta\leq 30^{\circ} \tag{7}$$

where Θ is the colatitude in the polar coordinates systems with the center of the eclipsed area as the pole. When we transform the coordinates to the geographical ones, we obtain

$$\Gamma(\theta)=\frac{1}{4}\{9b_0^2(\cos\theta_1+\sin\theta_1\cdot\cos(\lambda-\lambda_1)\cdot\tan\theta)^2-30b_0^4(\cos\theta_1$$
$$+\sin\theta_1\cdot\cos(\lambda-\lambda_1)\cdot\tan\theta)^4+25\,b_0^6(\cos\theta_1+\sin\theta_1\cdot\cos(\lambda-\lambda_1)\cdot\tan\theta)^6\} \tag{8}$$

Where θ_1 and λ_1 are the colatitude and longitude at t of the center of the geographical coordinates system and

$$b_0=\frac{1-\sin^2\theta\cdot\sin^2(\lambda-\lambda_1)}{1+\tan^2\theta\cos^2(\lambda-\lambda_1)}$$

The equation (6) can be written by the Fourier series and we obtain

$$\sigma=\sum_{s'=-\infty}^{\infty}f_{s'}\cos t \qquad \sigma^2=\sum_{s=-\infty}^{\infty}g_s\cos st \tag{9}$$

In general, the deformed conductivity in the ionosphere may be able to be always aproximated by an appropriate equation and we can treat it as the above. Thus, we can proceed similarly as the dynamo theory of Sq. Then, substituting (3) and (9) into (1) and simplifying, the left hand side of (1) reduces to

$$K^2\sum_m\sum_n\sum_{s=-\infty}^{\infty}g_s\sin\theta K_n^m\Big[\Big\{\frac{\partial H_s}{\partial\theta}\cdot\frac{dP_n^m}{d\theta}-n(n+1)H_sP_n^m\Big\}\sin[mt-\alpha_n^m]\cos st$$
$$+\frac{n}{\sin^2\theta}\frac{\partial H_s}{\partial\phi}P_n^m\cos[mt-\alpha_n^m]\mathrm{oos}st\Big] \tag{10}$$

It is, therefore, clear that R which must satisfy the equation (1) should be expressible as a series of surface harmonics,

$$R=\sum_m\sum_n K_n^m\cdot C\cdot K\sum_{N=0,}^{\infty}\sum_{M=-\infty}^{\infty}p_N^MP_N^M\sin(Mt-\alpha_N^M) \tag{11}$$

where p_N^M is a constant to be evaluated, and when M is a negative integer we assume that $P_N^M=P_N^{-M}$, After simplifications, the right hand side of equation (1),

$$CK^2\sum_m\sum_n\sum_{s'=-\infty}^{\infty}\sin\theta\cdot K_n^m\sum_M\sum_N p_N^M\Big[\Big\{\frac{Ms'}{\sin^2\theta}-N(N+1)\Big\}f_{s'}P_N^M$$
$$-\frac{\partial f_{s'}}{\partial\theta}\frac{dP_N^M}{d\theta}\Big]\sin[\overline{M+S'}t-\alpha_N^M\Big] \tag{12}$$

By using the relation

$$\frac{d}{d\theta}\sin\theta\frac{dP_N^M}{d\theta}+\sin\theta\Big[N(N+1)-\frac{M^2}{\sin^2\theta}\Big]P_N^M=0 \tag{13}$$

$$\sum_m\sum_n\sum_{s=-\infty}^{\infty} K_n{}^m g_s\Big[\frac{\partial H_z}{\partial\theta}\cdot\frac{dP_n{}^m}{d\theta}-n(n+1)H_zP_n{}^m\sin[(m+s)t-\alpha_n{}^m]$$
$$+\frac{n^2}{\sin\theta}\frac{\partial H_z}{\partial\phi}P_n{}^m\cos[(m+s)t-\alpha_n{}^m]\Big]$$
$$=-C\sum_m\sum_n K_n{}^m\sum_M\sum_N\sum_{s'=-\infty}^{\infty} p_N{}^M\Big[\Big\{N(N+1)-\frac{Ms'}{\sin^2\theta}\Big\}f_s/P_N{}^M+\frac{dfs'}{d\theta}\frac{dP_N{}^M}{d\theta}\Big]$$
$$\sin[(M+s')t-\alpha_N{}^M]=-C\sum_m\sum_n K_n{}^m\sum_M\sum_N\sum_{s'=-\infty}^{\infty} p_N{}^M R_N{}^M\sin[(M+s')t-\alpha_N{}^M] \qquad (14)$$

where

$$R_N{}^M=\Big[N(N+1)-\frac{Ms'}{\sin^2\theta}\Big]f_{s'}P_N{}^M+\frac{df_{s'}}{d\theta}\frac{dP_N{}^M}{d\theta} \qquad (15)$$

For any preassigned value of m and n, we have

$$-C\sum_M\sum_N\sum_{s=-\infty}^{\infty} p_N{}^M P_N{}^M\sin[(M+s')t-\alpha_N{}^M]$$
$$=\sum_{s=-\infty}^{\infty} g_s\Big[\Big\{\frac{\partial H_z}{\partial\theta}\frac{dP_n{}^m}{d\theta}-n(n+1)H_zP_n{}^m\Big\}\sin[(m+s)t-\alpha_n{}^m]$$
$$+\frac{m}{\sin^2\theta}\frac{\partial H_z}{\partial\phi}P_n{}^m\cos[(m+s)t-\alpha_n{}^m]\Big] \qquad (16)$$

Taking $H_z=C\cos\theta$, that is, assuming the coincidence of the axis of the rotation to the magnetic dipole axis,

$$\sum_M\sum_N\sum_{s'=-\infty}^{\infty} p_N{}^M R_N{}^M\sin[(M+s')t-\alpha_N{}^M]=\frac{1}{2n+1}\Big\{n(n+2)(n-m+1)P_{n+1}^m$$
$$+(n^2-1)(m+n)P_{n-1}^m\Big\}\sum_{s=-\infty}^{\infty} g_s\sin[(m+s)t-\alpha_n{}^m] \qquad (17)$$

Equating the coeffeicients of the corresqonding harmonic terms, we get

$$\sum_M\sum_N p_N{}^M R_N{}^M=\frac{1}{2n+1}\Big\{n(n+2)(n-m+1)P_{n+1}^m+(n^2-1)(m+n)P_{n-1}^m\Big\}g_s \qquad (17)$$

Equation (17) in general gives an infinite number of equations, which when solved gives $p_N{}^M$ and hence, current function R. [4] Reffering to the equation (15), it is evident the following is not always satisfied

$$p_N{}^M=p_{qN}^M-p_{qN}^M$$

Namely, even when the current function deduced from the dynamo equation for the σ_q and σ_e are added, the result is not always equal to the current function for the conductivity $\sigma(=\sigma_q-\sigma_e)$

§ 3. The magnetic field

The magnetic potential W at the surface of the earth of the radius r produced by the current function deduced above is given by the relations

$$W=-4\pi\sum_{N=0}^{\infty}\sum_{M=-N}^{N}\frac{N+1}{2N+1}\left(\frac{a}{r}\right)^{N}p_{N}^{M}P_{N}^{M}\sin[Mt-\alpha_{N}^{M}] \quad (18)$$

While, the magnetic potential of the Sq field Wq is given as follows,

$$W_{q}=-4\pi\sum_{N=0}^{\infty}\sum_{M=-N}^{N}\frac{N+1}{2N+1}\left(\frac{a}{r}\right)^{N}p_{qN}^{M}P_{N}^{M}\sin[Mt-\alpha_{N}^{M}] \quad (19)$$

Therefore, the effect on the Sq field can beyobtained from the magnetic potential

$$W_{e}=W_{q}-W \quad (20)$$

The position of the dynamo current is not known accurately, but, it is clear that the values of W does not depend very critically on a.

§ 4. Conclusion

It is shown, that the effects on the Sq field of the solar ecliqse can be deduced by the dynamo theory, even near the center of the Sq current system. In practical problems, computations may be simplified without affecting the results by neglecting the higher order terms. In paper, the geographical distributions of the deformed conductivity at one time point during the solar eclipse are considered, the time changes of the affected field at one station must be calculated at many time points, by repeatedly using of the equation. Expressing a locus of the zone of maximum eclipsed area explicitly by simple function of time t, and the effects may be calculated with only some alterations of the results.

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