Burst-like Occurrence of the Schumann Resonance

by

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Abstract

Sonogram of Schumann resonance shows a characteristic fine structure. The pattern of resonance line on sonogram is made of discrete spots. Each spot represents burst-like occurrence of oscillation. The peak frequency expressed by the ordinate of spot on sonogram fluctuates over the range of about 4 Hz. Theoretical considerations are made on the excitation and its response. The fluctuation of the peak frequency is considered to be due to variety of the excitation expressed by frequency function of flush current $I(\omega)$. In order to change the peak frequency from the resonance frequency determined by the frequency response function $A(\omega)$ of the earth-ionosphere cavity, high peak of the excitation $I(\omega)$ is necessary at a different ω. This is realized by multiple strokes of lightning flush.

1. Schumann resonance oscillations are the most conspicuous natural electromagnetic phenomena in the lower frequency side of ELF range. The oscillations are recognized as rather continuous waves on chart records which show amplitude versus time curve. Sonograms show continuous lines parallel to the time axis at the resonance frequencies usually. However the fine structure of sonogram is characterized by the crowd of discrete black spots. Fig. 1 shows the fine structure of Schumann resonances which were observed at Kakioka (36°14′N, 140°11′E; north magnetic field) and Memambetsu (43°55′N, 144°12′E; east electric field). One discrete spot expresses one individual burst-like occurrence of oscillation. The peak frequency ω_m at which the power of the field is maximum is given by the ordinate of the spot. Many spots occur in a short time interval, and they have different ordinates that are different peak frequencies. Even in the interval of 10 seconds or the less many different peak frequencies ω_m are observed.

Fig. 2 shows the occurrence-frequency of the peak frequency $f_m=\omega_m/2\pi$ around the first Schumann resonance frequency for north magnetic field at Kakioka. This is not power spectrum but it is the occurrence-frequency spectrum. The power of the oscillation is not taken into consideration to draw the occurrence-frequency curve except that the only spots having sufficient darkness to count are picked up.

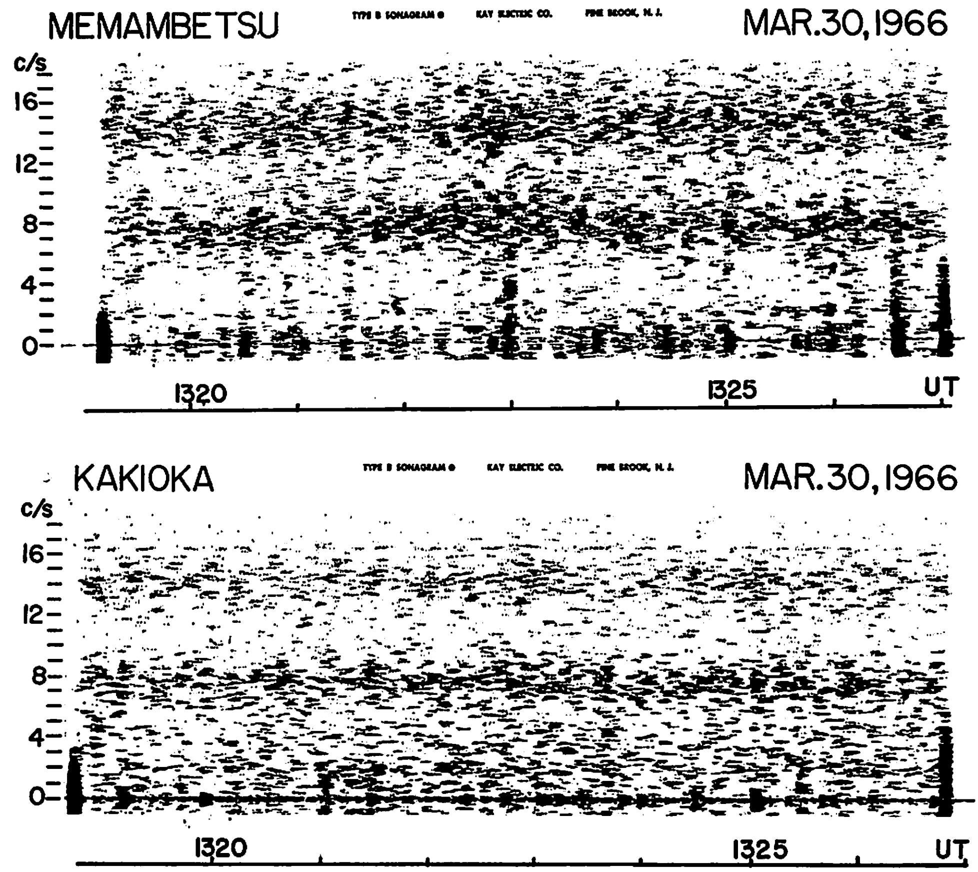

Fig. 1 Fine structure of Schumann resonances.

The maximum occurrence-frequency is 32/Hz/min. The half value 16/Hz/min occurs at 5.7 Hz and 9.7 Hz. Then the half value width is 4 Hz which is too wide to be explained by fluctuation of the Schumann resonance frequency. In addition, dispersion of the peak frequency occurs even in a short time interval such as 10 seconds, as it is seen in Fig. 1. Total number of the occurrence included in the half value width, that is the range between 5.7 Hz and 9.7 Hz, is 1.6/sec. This number is much smaller than 100 which is the estimated total number of lightning flush per second (Ishikawa, 1967).

The occurrence-frequency at frequencies lower than 5 Hz or higher than 10 Hz is not so low. This may be due to the other kind of burst-like oscillations such as upper atmospheric origin. Particularly in the lower frequency side probability

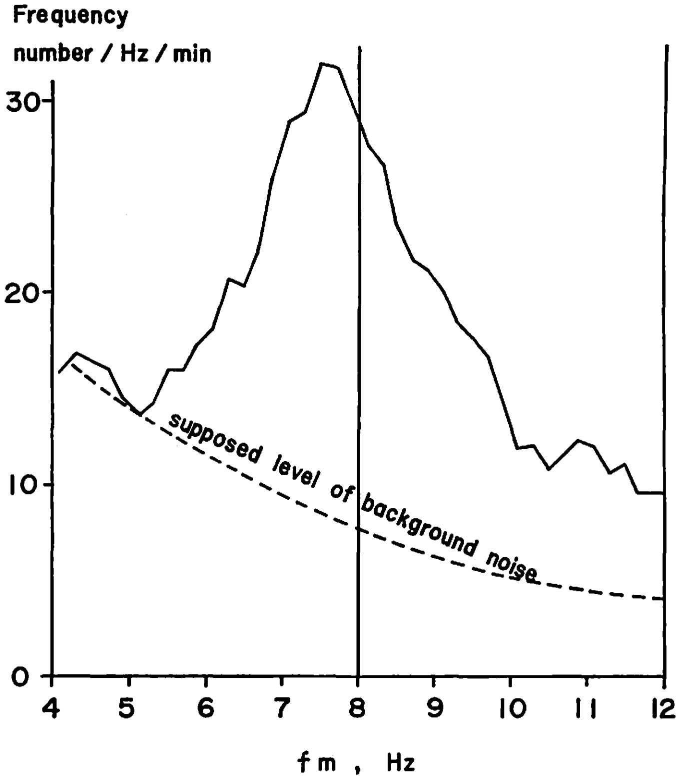

Fig. 2 Occurrence-frequency of the peak frequency f_m at which the power of field is maximum.

of such occurrence may be high. In the higher frequency side, which is the region between first and second Schumann resonances, each spots occur simultaneously at Kakioka and Memambetsu as it is seen in Fig. 1. The simultaneity is easily explained if they are considered to be caused by the same lightning flush though the peak frequency is nearly at the center of the first and second resonances. Here the level of occurrence-frequency shown by the broken line in Fig. 2 is assumed for background noises which are mainly made of natural phenomena of extra-ionospheric origin.

Near the resonance frequency some spots may be missed in counting because spots come closer or overlap each other with high darkness. Too low estimate of occurrence-frequency might be done at and near the resonance frequency.

2. In order to interpret theoretically the power spectrum of ELF, Galejs (1964) has used an ionosphere model of exponentially increasing conductivity and median lightning flush. He has got a rather satisfactory result for interpretation of the observed power spectrum. The observed power spectrum is expressed

generally by integrated power for a rather long time interval such as several minutes. The median lightning flush well represent the excitation in calculations of the integrated or mean power spectrum, because 10^4-10^5 flushes occur during each several minutes.

If one flush causes one burst of oscillation separately, the individual spectrum of flush current is to be taken into consideration to calculate the corresponding peak frequency at which the power is maximum.

The vertical electric field excited by a vertical electric dipole of length ds and current I in the earth-ionosphere cavity is given by (Wait, 1962; Galejs, 1964),

$$E=\frac{iIds\nu(\nu+1)}{4\pi\omega\mathcal{E}ha^2}\sum_{n=0}^{\infty}P_n(\cos\theta)\frac{2n+1}{n(n+1)-\nu(\nu+1)}, \tag{1}$$

where h is the height of the ionosphere and a is the radius of the earth. ν is the complex number which satisfy the equations,

$$kaS=\nu+\frac{1}{2} \tag{2}$$

and

$$k^2=\mathcal{E}\mu\omega^2-i\mu\omega\sigma, \tag{3}$$

where S is determined by the electrical character of the earth and ionosphere.

If excitation by a lightning flush current is expressed by

$$I(t)=\int_{-\infty}^{\infty}I(\omega)e^{i\omega t}d\omega, \tag{4}$$

the vertical electric field, as its response, is given by

$$E(t)=\int_{-\infty}^{\infty}A(\omega)I(\omega)e^{i\omega t}d\omega. \tag{5}$$

$A(\omega)$ is the frequency response function which is determined by the character of the earth-ionosphere cavity. Frequency dependency of $A(\omega)$ is not changed for such a short time as 10 seconds. Even if the difference of locality in flush occurrence is taken into consideration, fluctuation of the spectral peak of $A(\omega)$ occurs within the range of 1 Hz or the less. The diurnal variation of the resonance frequency shows the range of a few tenth of Hz (Ogawa, 1967). Then the fluctuation of the peak frequency shown in Figs. 1 and 2 is not due to the change of $A(\omega)$, but it is caused by the dissimilarity of $I(\omega)$ for each flushes.

A flush includes more than one strokes generally. The probability of multiple strokes is 75% (Kitagawa, 1966) or 85% (Galejs, 1964). When a flush is made of multiple strokes, maximum of $I(\omega)$ occurs at the frequency $f_m=\omega_m/2\pi=1/T$, where T is the interval of strokes.

If current of one stroke is expressed approximately by

$$I_1(t)=I_0te^{-\alpha t}, \tag{6}$$

$I_1(\omega)$ is given by

$$I_1(\omega)=I_0\left\{\frac{\alpha^2-\omega^2}{(\alpha^2+\omega^2)^2}-i\frac{2\alpha\omega}{(\alpha^2+\omega^2)^2}\right\}. \tag{7}$$

Considering that ω is in the Schumann resonance range,

$$|I_1(\omega)|^2\sim I_0^2/\alpha^4, \text{ for } \alpha\gg\omega. \tag{8}$$

Durations of stroke current are less than 1 m sec which is far shorter than the reciprocal of the Schumann resonance frequency. Even if the approximation of (6) is changed, $|I_1(\omega)|^2$ is nearly constant with respect to ω in the Schumann resonance range.

For multiple strokes, if each stroke current is the same, the frequency spectrum of flush current is given by

$$I_n(\omega)=I_1(\omega)\left(1+\sum_{k=2}^{n}e^{-i\omega\sum_{s=2}^{k}T_{s-1,s}}\right), \tag{9}$$

where n is the number of strokes and $T_{s-1,s}$ is the time interval between $(s-1)$th and s th strokes. If $T_{s-1,s}$ is constant and equal to T, $|I_n(\omega)|^2$ is given by,

$$|I_2(\omega)|^2=|I_1(\omega)|^2\times 2(1+\cos\omega T) \tag{10}$$

$$|I_3(\omega)|^2=|I_1(\omega)|^2\times(1+2\cos\omega T)^2 \tag{11}$$

$$|I_4(\omega)|^2=|I_1(\omega)|^2\times 8\cos^2\omega T\cdot(1+\cos\omega T) \tag{12}$$

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Fig. 3 shows the variation of $|I_n(\omega)|^2$ with respect to

$$\omega T/2\pi=(\omega/\omega_1)\beta, \tag{13}$$

where ω_1 is the first Schumann resonance frequency.

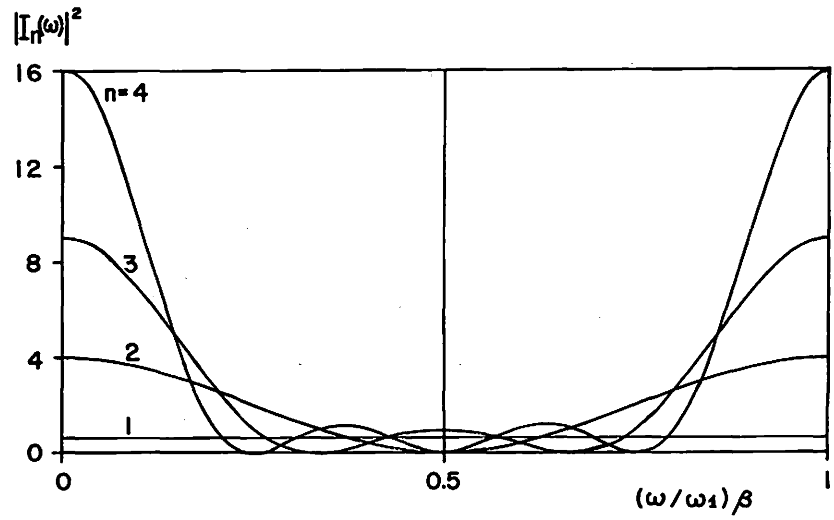

Fig. 3 Excitation $|I_n(\omega)|^2$ by multiple strokes. n is the number of stroke and ω_1 is the first Schumann resonance frequency. $\beta=T\cdot\omega_1/2\pi$.

When ELF caused by one flush is observed at one station, the frequency response function is given by

$$A(\omega)=\frac{\nu(\nu+1)(2n+1)}{\omega\{n(n+1)-\nu(\nu+1)\}}, \tag{14}$$

where the constant factor is omitted. In order to calculate the complex value ν, observed values of the resonance frequency f_n and Q_n (Balser and Wagner, 1960) shown in Table 1 is used. Using equations,

Table 1. Observed Schumann resonance frequency and Q (after Balser and Wagner)

	$n=1$	2	3	4	5
f_n in Hz	8.0	14.1	20.3	26.4	32.5
Q_n	4.0	4.5	5.0	5.5	6.0

$$ReS=(7.5/f_n)\{n(n+1)\}^{1/2} \tag{15}$$

and

$$ImS=(ReS)/2Q_n \tag{16}$$

and (2), (3) and (14), $|A(\omega)|^2$ is calculated. Fig. 4 shows the calculated $|A(\omega)|^2$ around the first Schumann resonance frequency ω_1.

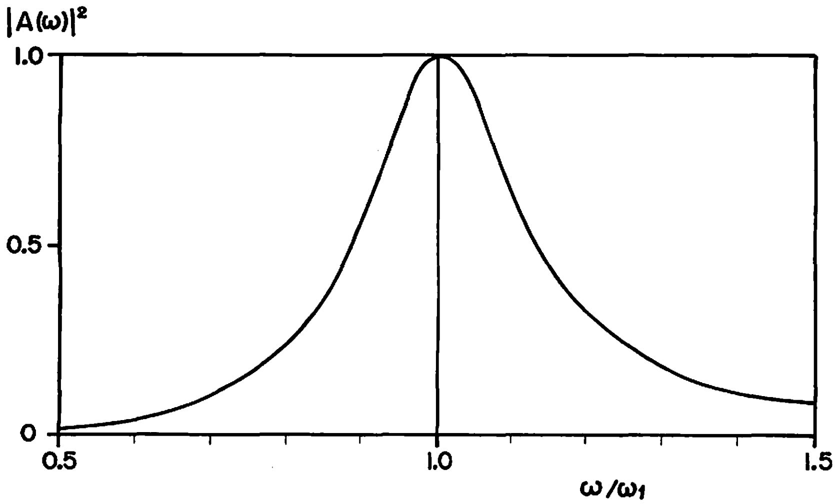

Fig. 4 Frequency response function $|A(\omega)|^2$ of the earth-ionosphere cavity.

The frequency response of power,

$$P_n(\omega)=|A(\omega)|^2\times|I_n(\omega)|^2 \tag{17}$$

is calculated from the values of $|A(\omega)|^2$ and $|I_n(\omega)|^2$ shown in Figs. 3 and 4 for each parameters n and T. The maximum of power, $P_n(\omega_m)$, and the corresponding frequency ω_m are shown in Fig. 5 with respect to $\beta=T\cdot\omega_1/2\pi=T\cdot f_1$ for the parameter $n=1$, 2, 3 and 4. When the stroke interval T is changed, ω_m/ω_1 is varied.

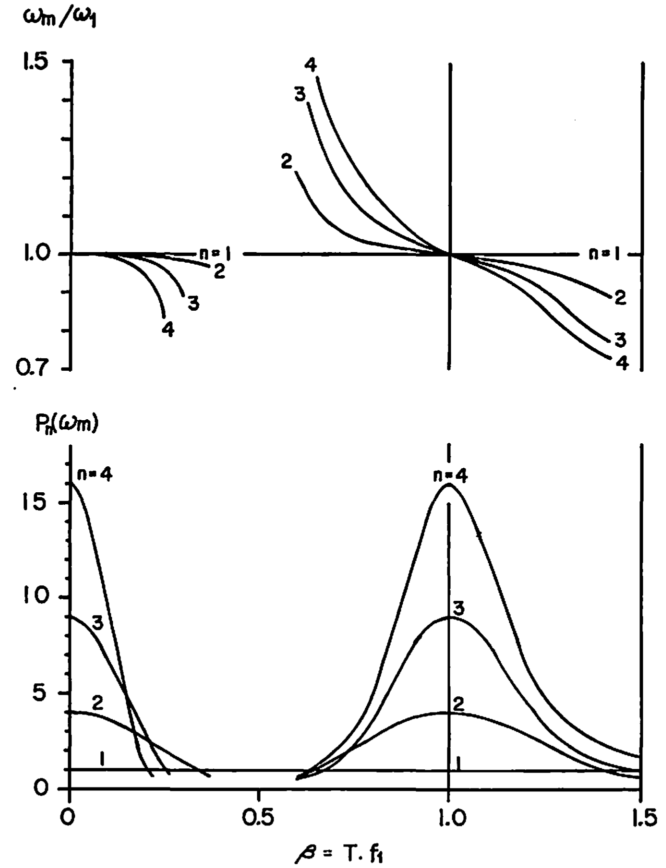

Fig. 5 Variations of maximum power $P_n(\omega_m)$ and the corresponding frequency ω_m with respect to $\beta=T\cdot f_1$.

The spot on sonogram appears at the frequency ω_m because the power is maximum at the frequency.

3. In actual case the number of stroke n and the stroke interval T is largely variable. Here the case of $n=3$ which is median value of stroke number is considered as a good example to calculate occurrence-frequency spectrum $F(\omega_m)$ of the peak frequency ω_m. For the occurrence-frequency of the stroke interval T Kitagawa's result (1966) is used. His histogram is rearranged with respect to $\beta=T\cdot f_1$ as it is shown in the lower half of Fig. 6. Multiplying the occurrence-frequency of stroke interval to the occurrence-frequency of ω_m/ω_1, $F(\omega_m)$ is calculated. In the upper half of Fig. 6 calculated $F(\omega_m)$ is shown for $n=3$.

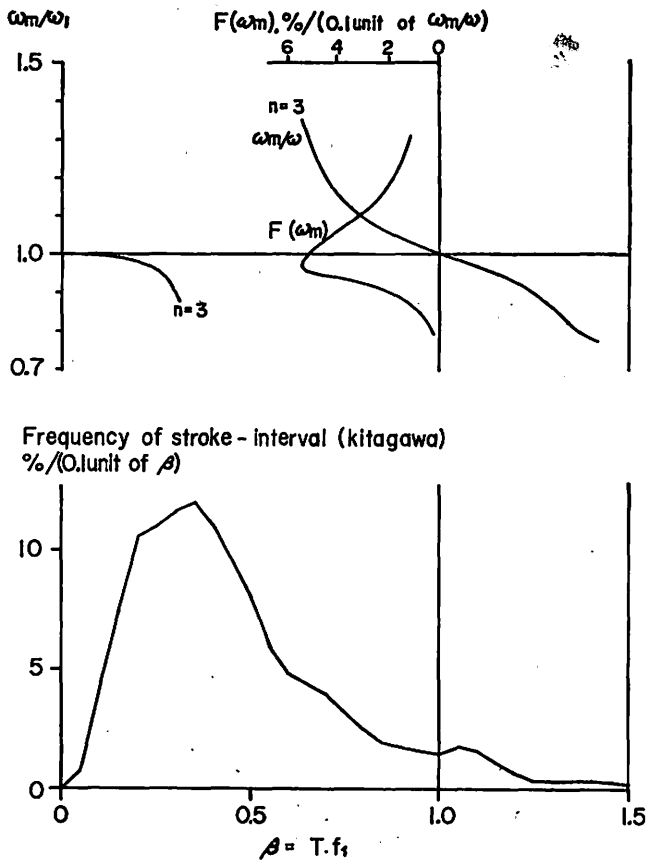

Fig. 6 Occurrence-frequency of stroke interval T (after Kitagawa) and calculated occurrence-frequency of ω_m, $F(\omega_m)$ for $n=3$.

Contribution from the small value of T is neglected in the $F(\omega_m)$ of Fig. 6. In the range of the smaller value of T, the maximum power $P_n(\omega)$ decreases quickly as T increases (Fig. 5). Occurrence-frequency of T, on the other hand, increases abruptly (Fig. 6). Then the contribution from the range to occurrence-frequency of spots with enough darkness to count in a sonogram depends largely on gradients of their increase and decrease. In spite of the neglection of the contribution from the smaller value of T, $F(\omega_m)$ shown in Fig. 6 is quite similar to the observed one.

If the contribution from the smaller values of T is effective, it must increase occurrence-frequencies at the resonance frequency f_1 and slightly smaller frequencies than f_1. This is compatible with the fact that the maximum of the observed occurrence-frequency occurs at a few tenths of Hz smaller than 8 Hz and probable missing of some spots around the resonance frequency in counting.

To calculate $F(\omega_m)$ several assumptions or approximations have been used. Some of them may not be suitable. For example, 1) theoretical calculations are made for vertical electric field though the observed data are magnetic or horizontal electric field, 2) distance and direction of source flushes are not taken into consideration, 3) spectrums are treated only around the first Schumann resonance frequency and 4) lightning flush statistics are not fully taken into consideration.

All modifications, however, will not change the principle that fluctuations of the peak frequency at which the power is maximum are due to the variety of the current spectrum of lightning flushes. And the variety is caused mainly by the change of the time interval of multiple strokes.

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シューマン共振振動のバースト状出現

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概 要

シューマン共振振動は連続した波のように見えるけれども，ソナグラムの微細構造を調べるとちりぢりになって点々状に分離している。すなわち点々の一つ一つがバースト状出現をあらわす。そしてその点々の示す周波数は 4 Hz 程度に分散している。この原因が雷放電の多重雷撃にあると考えて検討した。