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Periodic and Non-Periodic Components in Geomagnetic Secular Variation

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Abstract

Periodic component of geomagnetic secular variation is found when the non-periodic change is reduced from observed values. Its period and amplitude are often determined wrongly by careless estimate of the non-periodic component, such as simple approximation by a power function of time. The method of least squares should be applied to a sum of non-periodic and periodic terms. This is discussed in this paper. And then the analysis is made for 10 observatories' mean secular variation. Its period is about 60 years with the amplitude of 15−16 γ/yr in $\dot{H}$ or $\dot{D}$.

1. Introduction

Instrumental routine observation of geomagnetic field has been carried out for several ten years at many stations over the world. Geomagnetic secular variation is generally slow and non-periodic for a short time, but a few research workers have noted a periodic variation of several ten years period superposed upon the general tendency of slow change(1)(2). Dynamo-theoretical consideration also shows a possibility of oscillation of the earth's quadrupole whose period is 77 years(3).

Non-periodic component of the observed secular variation might be a part of a periodic variation with a much longer period which cannot be identified from such a short duration of the observation. However it is convenient to divide the variation into periodic and non-periodic components with respect to the duration of observation. Non-periodic variation is expressed by a linear sum of t^i, $\sum_{i=1}^{N} \beta_i t^i$, where t is time (expressed in unit of year in this paper) and N is generally not more than 2. For periodic component only periods more than a few ten years are considered in this paper. Variations with a shorter period, such as 11-year, may exist, but they are regarded as noises in the present study. In order to study such rapid variations, general trend of slow variation must be reduced. Hence one of the main purpose of this paper is to get accurate secular variation which includes both of periodic and non-periodic terms. After the accurate secular variation is reduced from observed values of geomagnetic field, significant rapid changes may be found in the residual. This last point will be discussed in another paper. Another purpose is to get oscillative mode of main geomagnetic field.

Secular variation is approximately expressed by $\sum_{i=1}^{N} \beta_i t^i$ for a short time. When extending this approximation to the whole period T_0 of observation, such as several ten years, it is rather difficult to directly obtain the correct values of the coefficients β_i's which express the real non-periodic term, because of superposition of periodic variation with period T near to T_0. First step to get the non-periodic term may be to calculate the best fit curve of $\sum_{i=1}^{N} \beta_i t^i$ for observed values during the period T_0 without regarding the periodic term. Thus obtained curve is clearly different from the real non-periodic component. This results in a different periodic term. Its period is significantly different from the real period T. It is approximately 3/4 of the observation period T_0, not depending upon the real period T (section 2).

It might be essentially difficult to identify a periodic term with a period which is near to the duration of observation. Nevertheless it is necessary to search for a better way to get more correct periodic term for the said study.

2. Apparent period

Apparent period which is usually seen in the secular variation is discussed first. Function $y(t)$ of time t is a sum of non-periodic and periodic terms expressed by,

$$y(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + a \sin(2\pi t/T) + b \cos(2\pi t/T) . \tag{1}$$

Values of $y(t)$ have been observed during $-T_0/2 \leqq t \leqq T_0/2$. The period T of the periodic term is near to the observation period T_0. The function $y(t)$ may be annual mean value of geomagnetic field (γ) or its rate of change (γ/yr), or some other quantities.

Supposing,

$$y_1(t) = [\beta_0] + [\beta_1]t + [\beta_2]t^2 \tag{2}$$

for first approximation of $y(t)$, best fit coefficients $[\beta_i]$'s are calculated by the least square method as follows.

$$\left.\begin{aligned} \sum y &= N[\beta_0] + \sum[\beta_1]t + \sum[\beta_2]t^2 \\ \sum yt &= \sum[\beta_0]t + \sum[\beta_1]t^2 + \sum[\beta_2]t^3 \\ \sum yt^2 &= \sum[\beta_0]t^2 + \sum[\beta_1]t^3 + \sum[\beta_2]t^4 . \end{aligned}\right\} \tag{3}$$

N is the total number of observation. If the observation is evenly distributed throughout the duration T_0 with sufficiently large N, summation $\sum$ in the right-hand side of the equation (3) can be replaced by integral and the terms of t and t^3 disappear. Considering that errors and noises in observed values of $y(t)$ are averaged and disappear in $\sum y$, $\sum yt$ and $\sum yt^2$, these summations are replaced by integral. Putting (1) into (3), the differences between the real and estimated coefficients,

$$\left.\begin{aligned}\beta_0-[\beta_0]&=\frac{15}{2}\left\{\frac{1}{5}\frac{p}{\pi}\sin\frac{\pi}{p}-\left(\frac{p}{\pi}\right)^2\left(\frac{p}{\pi}\sin\frac{\pi}{p}-\cos\frac{\pi}{p}\right)\right\}b=bf_0(p)\\ \beta_1-[\beta_1]&=-\frac{1}{T_0}\frac{p}{\pi}\left\{\frac{p}{\pi}\sin\frac{\pi}{p}-\cos\frac{\pi}{p}\right\}a=af_1(p)/T_0\\ \beta_2-[\beta_2]&=\frac{30}{T_0^2}\left\{-\frac{p}{\pi}\sin\frac{\pi}{p}+3\left(\frac{p}{\pi}\right)^2\left(\frac{p}{\pi}\sin\frac{\pi}{p}-\cos\frac{\pi}{p}\right)\right\}b=bf_2(p)/T_0^2\end{aligned}\right\}\quad(4)$$

are expressed by functions of p, which is

$$p=T/T_0\,. \qquad (5)$$

Residual part of $y(t)$,

$$\Delta y_1(t)=y(t)-y_1(t) \qquad (6)$$

still includes non-periodic term with the coefficients given by the equation (4). It gives an apparent periodic variation with an apparent period much different from the real period T for $-T_0/2\leqq t\leqq T_0/2$. Examples are shown in Fig. 1, whose upper part shows two cases of $y(t)$ with

(a) $\beta_0=\beta_2=b=0,\quad \beta_1=-5.33/T_0,\quad a=1,\quad T=(3/2)T_0\,,$

(b) $\beta_0=\beta_2=b=0,\quad \beta_1=-3.60/T_0,\quad a=1,\quad T=T_0\,.$

Three straight lines of each $y(t)$ express the real non-periodic term $\beta_1 t$, the first approximation $[\beta_1]t$ and their average $([\beta_1]+\beta_1)t/2$, respectively. Lower three curves in each column are residuals after subtracting the indicated non-periodic variation from $y(t)$. Both $\Delta y_1(t)$ shown in the top of them gives an apparent period very near to $(3/4)T_0$, without distinction of the real period T, which is $(3/2)T_0$ or T_0. The lowest curve expresses the real periodic term, and the middle shows again an apparent period moderately different from the real one.

Residual curves, such as those shown in Fig. 1, include non-periodic term so that they are quasi-periodic in the given internal T_0. Determination of their period is generally difficult, particularly in the actual case which includes noises in the observed value of $y(t)$. Here apparent period T_a is defined as it is the interval between the points where the residual is zero (Fig. 1). T_a depends upon the determined coefficients of non-periodic term in $y(t)$. For $y(t)=\beta_1 t+a\sin(2\pi t/T_0)$, change of T_a in the residual function $\Delta y_1'(t)=-\Delta\beta_1 t+a\sin(2\pi t/T_0)$ is shown in Fig. 2 with respect to error $\Delta\beta_1$ in determination of β_1. Abscissa of the figure shows $\Delta\beta_1$ in unit of $(\beta_1-[\beta_1])$. The origin of the abscissa represents $\Delta\beta_1=0$, hence $T_a=T_0$, and $\Delta\beta_1=-1.0(\beta_1-[\beta_1])$ is the case of the said first approximation, $\Delta y_1(t)=y(t)-y_1(t)$. For $\Delta\beta_1>0$, the point of zero residual is not located within the interval $-T_0/2\leqq t\leqq T_0/2$,

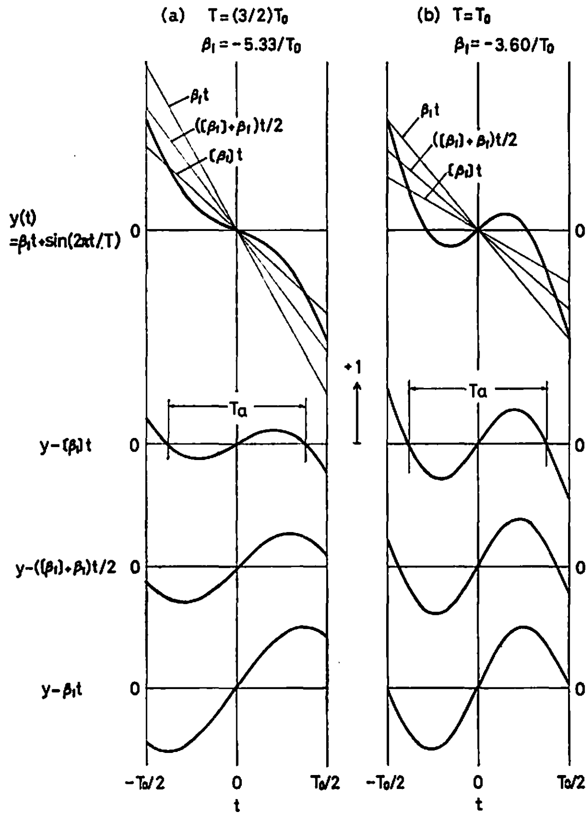

Fig. 1. Examples showing how the apparent period of residual function differs from the real period T depending upon the non-periodic term subtracted from the original variation $y(t)$ which is the sum of non-periodic term, $\beta_1 t$ and periodic one, $\sin(2\pi t/T)$.

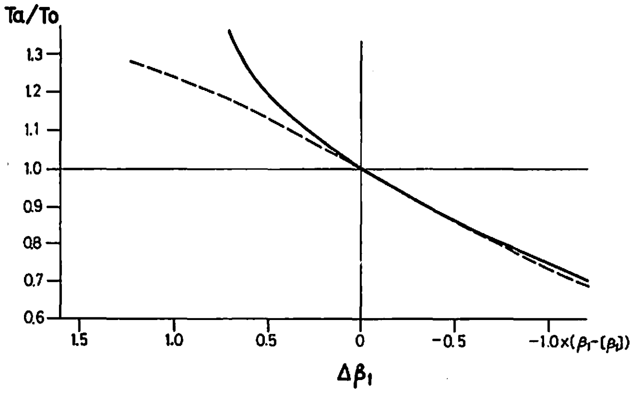

Fig. 2. Change of the apparent period T_a which is due to the error $\Delta\beta_1$ in dermination of the coefficient β_1 for $y(t)=\beta_1 t+a\sin(2\pi t/T_0)$.

then T_a is not determined. But extending the theoretical function outside the interval, T_a is calculated and shown for reference.

Broken line in the figure shows a more correct period which is obtained in a reasonable way described in the next section. Tow lines nearly coincide with each other in the region of $\Delta\beta < 0$. T_a obtained in the said simple way gives nearly correct value of the period of the residual quasi-periodic variation, which is not the period T of the original variation $y(t)$. It nearly sticks in $(3/4)T_0$ for the given examples of $\Delta y_1(t)$ whose original functions $y(t)$ are anti-symmetric with respect to $t = 0$. Reason why only anti-symmetric terms are considered will be described in the next section.

T_a of the anti-symmetric term of $\Delta y_1(t)$ is calculated from the interval between two t values which satisfy the equation,

$$f_1(p)(t/T_0) + \sin(2\pi t/pT_0) = 0,$$

and it changes shightly with $p(=T/T_0)$ as it is shown in Fig. 3. Therefore the original

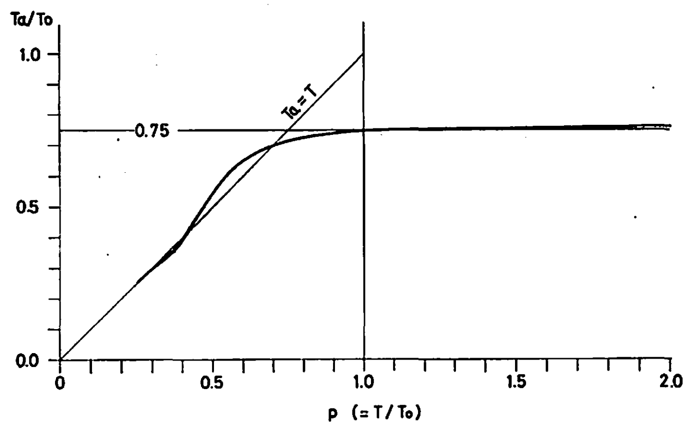

Fig. 3. The apparent period T_a of anti-symmetric term of $\Delta y_1(t)$, which varies with the period T in the original function $y(t)$.

period T may be determined from T_a. As T decreases, T_a approaches to T. For shorter T, period determination is easy. For longer T it is not so easy to distinguish slight change of T_a, particulary for $T > T_0$. This difficulty is shown in another example of Fig. 4, whose three curves are very similar in the interval $-T_0/2 \leqq t \leqq T_0/2$. Each curve represents anti-symmetric function $y(t) = \beta_1 t + a\sin(2\pi t/T)$ with different period T, $T = (3/2)T_0$, T_0 or $(3/4)T_0$ and with different confficients β_1 and a which are chosen so as to give quite similar curve in the interval $-T_0/2 \leqq t \leqq T_0/2$. From

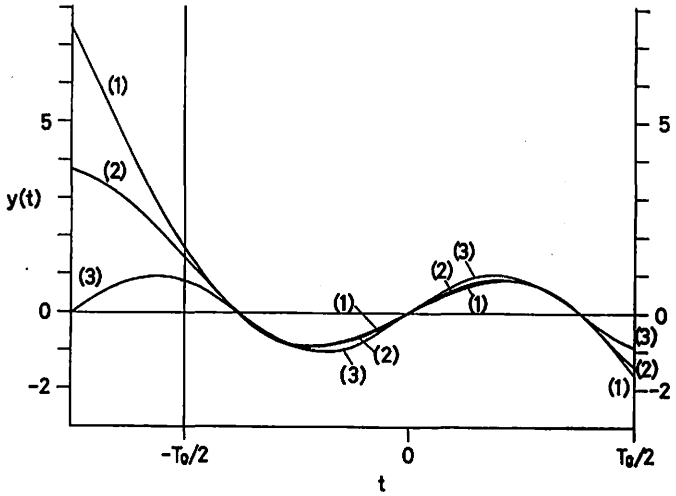

Fig. 4. Very similar curves in the interval $-T_0/2 \leqq t \leqq T_0/2$.
Each curve represents $y(t)=\beta_1 t + a\sin(2\pi t/T)$, with
(1) $\beta_1=-10.20/T_0$, $a=3.89$, $T=(3/2)T_0$
(2) $\beta_1=-2.97/T_0$, $a=1.55$, $T=T_0$
(3) $\beta_1=0$, $a=1$, $T=(3/4)T_0$

observed values in this interval, it may be very difficult to distinguish the difference between the curves, though their periods are different very much from each other.

Even though the inevitable uncertainty is included, $T-T_a$ curve is a simple but valuable way to estimate the real period T, compared with misunderstanding that an apparent period, such as T_a, is supposed to be the real period.

3. Analysis of period

Usual method of spectral analysis is not applicable to the variation observed within the limited interval T_0 which is near to the period T concerned. In order to get the period of $\Delta y_1(t)$, the principle of the least square error is here used. The function $\Delta y_1(t)$ is divided into two parts, anti-symmetric function $\Delta y_{1a}(t)$ and symmetric function $\Delta y_{1s}(t)$,

$$\left.\begin{aligned} \Delta y_{1a}(t) &= \{\Delta y_1(t) - \Delta y_1(-t)\}/2 \\ \Delta y_{1s}(t) &= \{\Delta y_1(t) + \Delta y_1(-t)\}/2 \end{aligned}\right\} \qquad (7)$$

for the sake of convenience.

If original $y(t)$ is expressed by (1) without regarding noises or errors in the observation, those divided parts of $\Delta y_1(t)$ is given by,

$$\left.\begin{aligned}\Delta y_{1a}(t) &= a\{f_1(p)t/T_0 + \sin(2\pi t/pT_0)\}\\ \Delta y_{1s}(t) &= b\{f_0(p) + f_2(p)(t/T_0)^2 + \cos(2\pi t/pT_0)\}\,.\end{aligned}\right\} \tag{8}$$

Only one parameter p is included in these expression except the amplitude a or b of the original periodic term.

For anti-symmetric function $\Delta y_{1a}(t)$, amplitude $a_1(p_1)$ of the spectral component $a_1(p_1)\sin(2\pi t/p_1T_0)$ is determined for a given period T_1,

$$T_1 = p_1T_0\,, \tag{9}$$

so as to make integrated square error,

$$\int_{-T_0/2}^{T_0/2}\{\Delta y_{1a}(t) - a_1(p_1)\sin(2\pi t/p_1T_0)\}^2dt$$

minimum. Namely,

$$a_1(p_1) = \int_{-T_0/2}^{T_0/2}\Delta y_{1a}(t)\sin(2\pi t/p_1T_0)\,dt\Big/\int_{-T_0/2}^{T_0/2}\sin^2(2\pi t/p_1T_0)\,dt\,. \tag{10}$$

Using the minimum integrated square error, spectrum $S_{1a}(p_1)$ is defined here by,

$$S_{1a}(p_1) = 1 - \int_{-T_0/2}^{T_0/2}\{\Delta y_{1a}(t) - a_1(p_1)\sin(2\pi t/p_1T_0)\}^2\,dt\Big/\int_{-T_0/2}^{T_0/2}\{\Delta y_{1a}(t)\}^2\,dt \tag{11}$$

Similarly, amplitude $b_1(p_1)$ and spectrum $S_{1s}(p_1)$ of symmetric function $\Delta y_{1s}(t)$ are given by,

$$b_1(p_1) = \int_{-T_0/2}^{T_0/2}\Delta y_{1s}(t)\cos(2\pi t/p_1T_0)\,dt\Big/\int_{-T_0/2}^{T_0/2}\cos^2(2\pi t/p_1T_0)dt \tag{12}$$

and,

$$S_{1s}(p_1) = 1 - \int_{-T_0/2}^{T_0/2}\{\Delta y_{1s}(t) - b_1(p_1)\cos(2\pi t/p_1T_0)\}^2\,dt\Big/\int_{-T_0/2}^{T_0/2}\{\Delta y_{1s}(t)\}^2\,dt\,, \tag{13}$$

respectively. Putting (8) into (10)−(13), $a_1(p_1)/a$, $b_1(p_1)/b$, $S_{1a}(p_1)$ and $S_{1s}(p_1)$ are expressed by functions of p_1 with a parameter p,

$$\begin{aligned}a_1(p_1)/a = 2\Big\{&\frac{p_1f_1(p)}{\pi}\Big(\sin\frac{\pi}{p_1} - \frac{\pi}{p_1}\cos\frac{\pi}{p_1}\Big) + \frac{p}{p_1-p}\sin\Big(\frac{\pi}{p} - \frac{\pi}{p_1}\Big)\\ &- \frac{p}{p_1+p}\sin\Big(\frac{\pi}{p} + \frac{\pi}{p_1}\Big)\Big\}\Big/\Big(\frac{2\pi}{p_1} - \sin\frac{2\pi}{p_1}\Big)\end{aligned} \tag{14}$$

$$\begin{aligned}b_1(p_1)/b = 2\Big[&\Big\{2f_0(p) + \frac{1}{2}f_2(p)\Big\}\sin\frac{\pi}{p_1} - \frac{1}{\pi^2}f_2(p)\Big(\sin\frac{\pi}{p_1} - \frac{\pi}{p_1}\cos\frac{\pi}{p_1}\Big)\\ &+ \frac{p}{p_1-p}\sin\Big(\frac{\pi}{p} - \frac{\pi}{p_1}\Big) + \frac{p}{p_1+p}\sin\Big(\frac{\pi}{p} + \frac{\pi}{p_1}\Big)\Big]\Big/\Big(\frac{2\pi}{p_1} + \sin\frac{2\pi}{p_1}\Big)\end{aligned} \tag{15}$$

$$S_{1a}(p_1)=\left\{\frac{a_1(p_1)}{b}\right\}^2\left(2\pi-p_1\sin\frac{2\pi}{p_1}\right)\Big/\left[\frac{\pi}{3}\{f_1(p)\}^2+\frac{4}{\pi}f_1(p)\left(\sin\frac{\pi}{p}-\frac{\pi}{p}\cos\frac{\pi}{p}\right)\right.$$
$$\left.+2\pi-p\sin\frac{2\pi}{p}\right] \tag{16}$$

$$S_{1s}(p_1)=\left\{\frac{b_1(p_1)}{b}\right\}^2\left(2\pi+p_1\sin\frac{2\pi}{p_1}\right)\Big/\left[4\pi\{f_0(p)\}^2+\frac{2}{3}\pi f_0(p)f_2(p)+\frac{\pi}{20}\{f_2(p)\}^2\right.$$
$$\left.+8f_0(p)p\sin\frac{\pi}{p}+\frac{2}{\pi^2}f_2(p)\left\{\frac{2\pi}{p}\cos\frac{\pi}{p}+\left(\frac{\pi^2}{p^2}-2\right)\sin\frac{\pi}{p}\right\}+2\pi+p\sin\frac{2\pi}{p}\right]. \tag{17}$$

These are theoretical function when neglecting noises or errors in $y(t)$ observation.

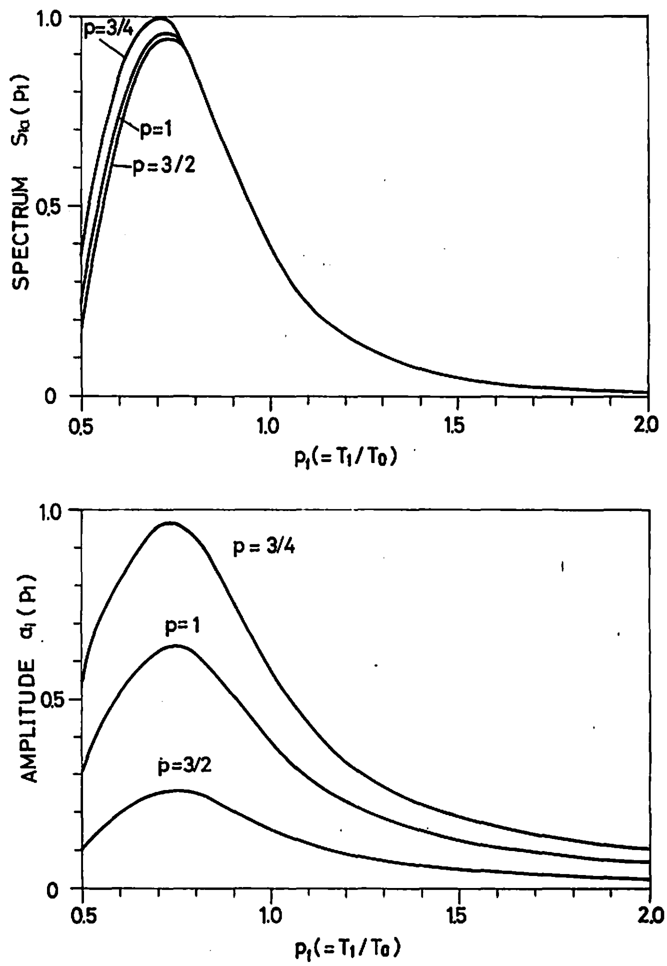

Fig. 5. Spectrum $S_{1a}(p_1)$ and amplitude $a_1(p_1)$ of $\Delta y_{1a}(t)$.

Figs. 5 and 6 show the theoretical curves calculated from (14)—(17) for parameters $p=3/4$, 1 and 3/2.

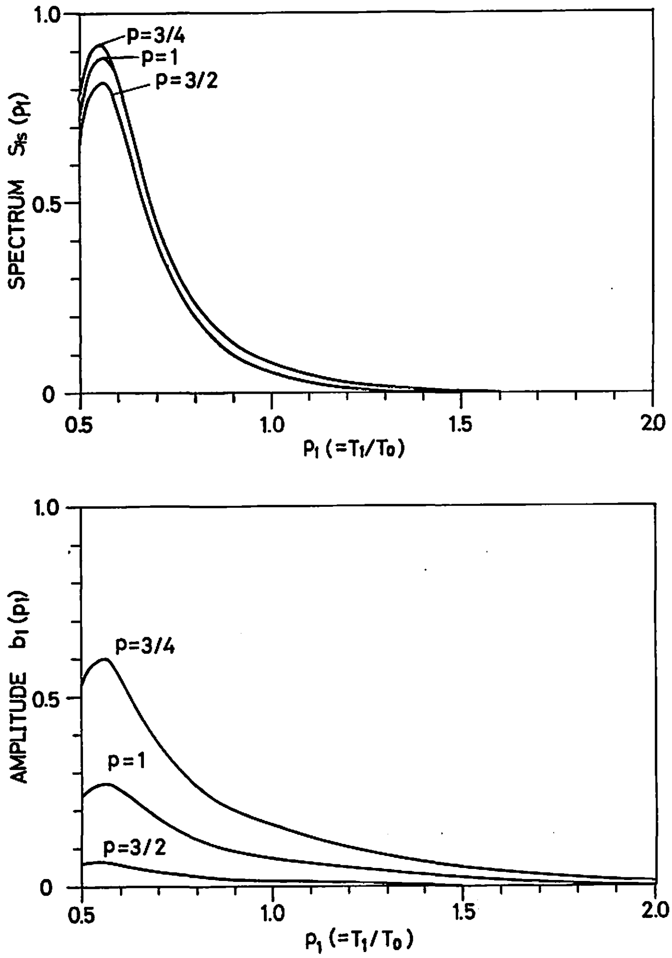

Fig. 6. Spectrum $S_{1s}(p_1)$ and amplitude $b_1(p_1)$ of $\Delta y_{1s}(t)$.

The spectrum $S_{1a}(p_1)$ or $S_{1s}(p_1)$ here defined may not give the real spectrum in usual meaning. But it gives the best fit periodic variation in the interval $-T_0/2 \leqq t \leqq T_0/2$ according to the definition. The spectral peak gives the best fit period of $\Delta y_{1a}(t)$ or $\Delta y_{1s}(t)$ and the shape of the spectral curve shows something like spectral structure. The best fit period of $\Delta y_{1a}(t)$ or $\Delta y_{1s}(t)$ does not change so much even if the original period $T(=pT_0)$ changes very much, as it is seen in Fig. 5 or 6. It depends rather upon the observation period T_0. This is a quite similar result as that of apparent period T_a.

If some other functions are used for $\Delta y_{1a}(t)$ or $\Delta y_{1s}(t)$, their spectrum can be calculated similarly. When the coefficient of an approximate function of the non-periodic term is β'_i for $[\beta_i](i=0$, 1 and 2), $f_0(p)$, $f_1(p)$ and $f_2(p)$ should be replaced by $(\beta_0-\beta_0')/b$, $(\beta_1-\beta_1')T_0/a$ and $(\beta_2-\beta_2')T_0^2/b$, respectively, in (14), (15), (16) and (17). The best fit period of thus obtained spectral peak is a function of $\Delta\beta_i=\beta_i'-\beta_i$, which is the error in determination of non-periodical term of the original function $y(t)$, for a given parameter $p(=T/T_0)$. An example of the period variation is shown in Fig. 2 (broken line) for $\Delta y'_{1a}(t)=-\Delta\beta_1 t+a\sin(2\pi t/T_0)$.

Using the theoretical spectrum $S_{1a}(p_1)$ or $S_{1s}(p_1)$, best fit terms of observed $y(t)$ are obtained as follows. From observed values of $y(t)$, $[\beta_i]$'s are calculated according to (3). Then observed $\Delta y_{1a}(t)$ and $\Delta y_{1s}(t)$ are obtained by using (2), (6) and (7). Putting them into (10)$-$(13), observed spectrum $S_{1a}(p_1)$ and $S_{1s}(p_1)$ are calculated. Comparing these observed spectrums with theoretical ones, such as those given in Fig. 5 and 6, best fit parameter p is determined. In this comparison both spectrums should be normalized because the peak of the observed spectrum may be lowered by noises in observed values. The determined p gives $f_i(p)$, which is put into (8). Then the least square method gives the best fit value of a or b from observed values of $\Delta y_{1a}(t)$ or $\Delta y_{1s}(t)$ and calculated values of the right-hand side of (8).

Choice of the best fit parameter p is not so easy, particularly for $p>1$ $(T>T_0)$, because the difference between spectral curves is not so much. This difficulty is unavoidable anyway as it is described in preceding sections.

In actual application, use of anti-symmetric function is convenient to determine p unless the phase of the periodic variation is close to cosine term, because some parts of the cosine term are extracted away by the first approximation of $\beta_0+[\beta_2]t^2$. Similar effect of $[\beta_1]t$ may be found in sine term too, but it must be smaller than that in cosine term.

Actual examples of application are shown in section 5.

4 Direct determination of periodic and non-periodic terms

Analysis of period in the preceding section has started from $\Delta y_1(t)$ under the condition that the first approximation of non-periodic term was made before. And then the original terms in $y(t)$ is obtained from the spectrum of $\Delta y_1(t)$. Leading principle in the procedure is the least square error. If so, direct application of the same principle to the original $y(t)$ may be simpler.

Supposing,

$$y_2(t)=\beta_0(p_2)+\beta_1(p_2)t+\beta_2(p_2)t^2+a_2(p_2)\sin(2\pi t/p_2T_0)+b_2(p_2)\cos(2\pi t/p_2T_0), \quad (18)$$

for the best fit expression of observed $y(t)$, coefficients $\beta_0(p_2)$, $\beta_1(p_2)$, $\beta_2(p_2)$, $a_2(p_2)$, $b_2(p_2)$ and p_2 are determined so as to make the integrated square error,

$$\int_{-T_0/2}^{T_0/2} \{y(t) - y_2(t)\}^2 dt$$

minimum, where

$$p_2 = T_2/T_0 . \tag{19}$$

When $y(t)$ is exactly the same as given by (1), $\beta_0(p_2) = \beta_0$, $\beta_1(p_2) = \beta_1$, $\beta_2(p_2) = \beta_2$, $a_2(p_2) = a$, $b_2(p_2) = b$ and $p_2 = p(T_2 = T)$.

It is convenient to divide $y_2(t)$ into symmetric and anti-symmetric terms because of the same reason as is described for $y_1(t)$ in the previous section. Here anti-symmetric function,

$$y_{2a}(t) = \beta_1(p_2)t + a_2(p_2)\sin(2\pi t/p_2T_0) \tag{20}$$

is considered first. Corresponding observed value of anti-symmetric term is obtained from,

$$y_a(t) = \{y(t) - y(-t)\}/2 . \tag{21}$$

From the observed values of $y(t)$, the best fit coefficients of the anti-symmetric term are calculated by,

$$\left.\begin{aligned} a_2(p_2) &= \frac{N(p_2)\int_{-T_0/2}^{T_0/2} y(t)\,dt - L(p_2)\int_{-T_0/2}^{T_0/2} y(t)\sin(2\pi t/p_2T_0)dt}{\{N(p_2)\}^2 - L(p_2)M(p_2)} \\ \beta_1(p_2) &= \frac{N(p_2)\int_{-T_0/2}^{T_0/2} y(t)\sin(2\pi t/p_2T_0)dt - M(p_2)\int_{-T_0/2}^{T_0/2} y(t)t\,dt}{\{N(p_2)\}^2 - L(p_2)M(p_2)} \end{aligned}\right\} \tag{22}$$

for a given p_2, where,

$$\left.\begin{aligned} L(p_2) &= \int_{-T_0/2}^{T_0/2} t^2\,dt \\ M(p_2) &= \int_{-T_0/2}^{T_0/2} \sin^2(2\pi t/p_2T_0)\,dt \\ N(p_2) &= \int_{-T_0/2}^{T_0/2} t\sin(2\pi t/p_0T_0)\,dt . \end{aligned}\right\} \tag{23}$$

Using the minimum integrated square error of the anti-symmetric term for a given p_2,

$$\int_{-T_0/2}^{T_0/2} \{y_a(t) - y_{2a}(t)\}^2 dt ,$$

spectrum $S_{2a}(p_2)$ is similarly defined by,

$$S_{2a}(p_2) = 1 - \int_{-T_0/2}^{T_0/2} \{y_a(t) - y_{2a}(t)\}^2 dt \bigg/ \int_{-T_0/2}^{T_0/2} \{y_a(t)\}^2 dt$$
$$= \frac{L(p_2)\{\beta_1(p_2)\}^2 + M(p_2)\{a_2(p_2)\}^2 + 2N(p_2)\beta_1(p_2)a_2(p_2)}{\int_{-T_0/2}^{T_0/2} \{y_a(t)\}^2 dt}. \quad (24)$$

Peak of the spectrum determines the best fit period p_2, which then gives the best fit $a_2(p_2)$ and $\beta_1(p_2)$ by (22).

For symmetric function, similar way must give β_0, β_2 and p_2 which is not necessarily equal to the best fit p_2 obtained from anti-symmetric function. However accuracy in determining the period may be lower in symmetric function than in anti-symmetric one unless the phase of the periodic term is close to the cosine term, as it is described in the previous section. In actual application, p_2, so the period, is determined from anti-symmetric term, and then symmetric term is calculated by the least square method using the determined p_2.

All values of the coefficients are rigorously determined by this method in mathematical meaning. However, reliability of the determined period, and then of all the determined values, is not so different from that in the method described in the preceding sections, because the spectrum $S_{2a}(p_2)$ shows very broad peak as it is shown in the actual example of the next section (see Fig. 10). And each calculation in this method needs a lot of figures for each term to detect slight difference in spectral value and to treat unprocessed values of $y(t)$.

Table 1. List of observatories

Observatory	Geomagnetic		Preceding or Supplementary observatory	Period
	Latitude	Longitude, E		
San Juan	29.6	3.1	Vieques	1903–
Chambon-la-Foret	50.5	84.4	Val Joyeux	1901–
Misallat	26.9	105.9	Helwan, Ksara	1903–
Vysokaya Dubrava	48.5	140.7	Sverdlovsk	1900–
Yangi-Bazar	32.3	144.0	Keles, Dehra Dun	1903–
Patrony	40.7	174.7	Irkutsk, Zuy	1900–
Kakioka	26.0	206.0	Zi-Ka-Wei, Lukiapan	1900–
Honolulu	21.1	266.5		1902–
Tucson	40.4	312.2		1910–
Fredericksburg	49.6	349.8	Cheltenhum	1901–

5. Geomagnetic secular variation in recent 65 years

Geomagnetic secular variations are various in the world, but periodical terms are not so different from each other. Here their zonal terms are studied for examples of application of the method described above. Data are annual mean values(4),(5),(6) of geomagnetic field from 1900 to 1965 at 10 observatories which are distributed rather evenly in the middle latitude zone from 20° to 50° in geomagnetic latitude. Those are San Juan, Chambon-la-Foret, Misallat, Vysokaya-Dubrava, Yangi-Bazar, Patrony, Kakioka, Honolulu, Tucson and Fredericksburg (Table 1). The observatory names are those in 1965 and the preceding observatories are indicated in the table.

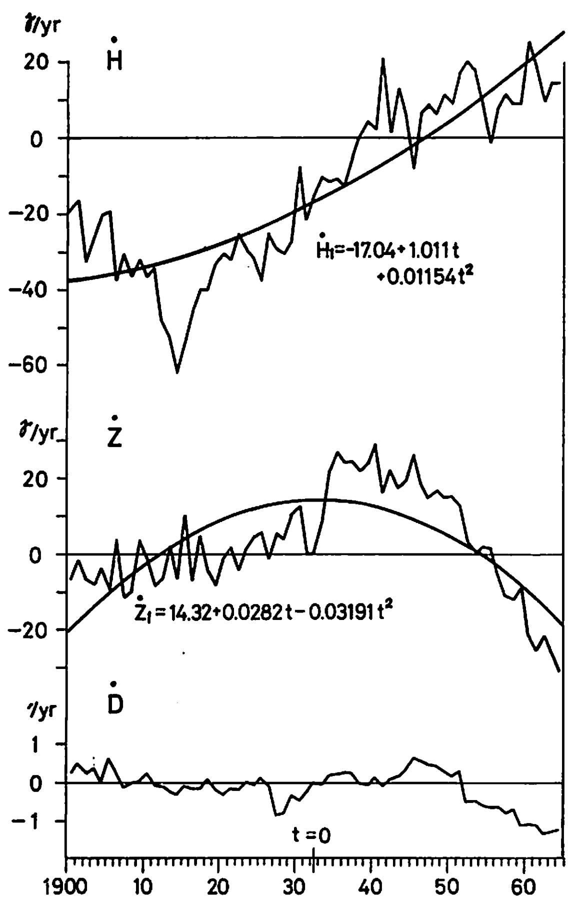

Fig. 7. Ten observatories' mean of annual mean rate of change in three components of geomagnetic field, $\dot{H}$, $\dot{Z}$ and $\dot{D}$. First approximations $\dot{H}_1$ and $\dot{Z}_1$ of non-periodic term also are shown for $\dot{H}$ and $\dot{Z}$.

Ten observatories' mean of annual mean rate of change in three components, $\dot{H}$, $\dot{Z}$ and $\dot{D}$, are calculated (Fig. 7). For periods of no record in an observatory, data of a near-by observatory, which is indicated in Table 1, are supplementarily used. If this is impossible, simple interporation is carried out. For the first decade from 1900 to 1910, as some observatories had not been operated yet, means for existing observatories' data are calculated. All the supplementary values are corrected to be connected smoothly to the values before and after the period concerned by using the difference obtained when the same procedure is extended before and after.

As $\dot{H}$, $\dot{Z}$ and $\dot{D}$ of Fig. 7 are ten observatories' mean, they express the zonal part. Change of $\dot{D}$ is small. $\dot{H}$ or $\dot{Z}$ shows superposition of a periodical term upon a non-

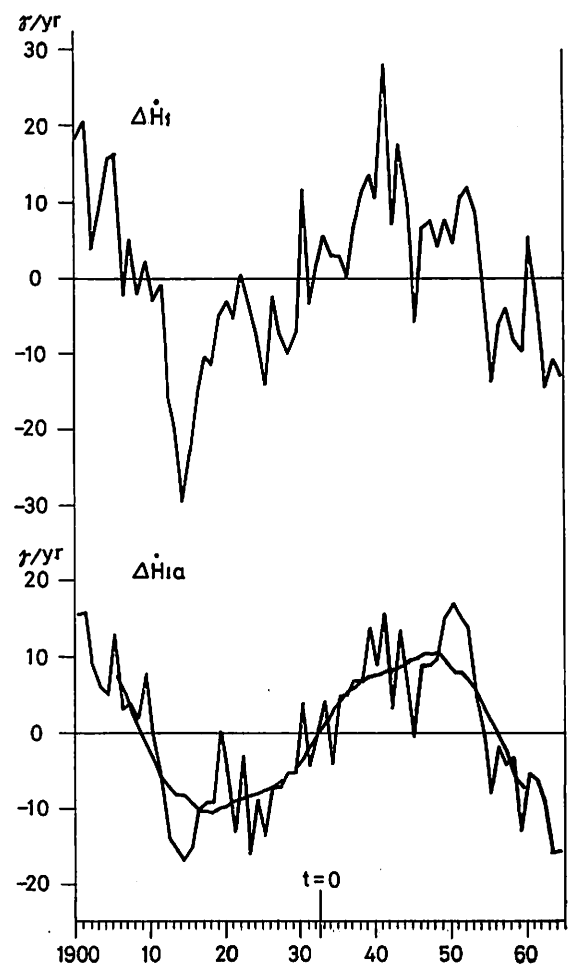

Fig. 8. Residual $\Delta\dot{H}_1$ and its anti-symmetric term $\Delta\dot{H}_{1a}$ 11-year running average also is shown for $\Delta\dot{H}_{1a}$.

periodical variation. $\dot{H}$ is considered first for $y(t)$ in the preceding sections. First approximation of $\dot{H}$,

$$\dot{H}_1 = -17.04 + 1.011t + 0.01154t^2 \tag{25}$$

is obtained from observed values shown in Fig. 7. Fig. 8 shows $\Delta\dot{H}_1 = \dot{H} - \dot{H}_1$ and its antisymmetric term $\Delta\dot{H}_{1a}(t) = \{\Delta\dot{H}_1(t) - \Delta\dot{H}_1(-t)\}/2$. Smooth curve of $\Delta\dot{H}_{1a}$ is the 11-year running average, which is calculated to determine the apparent period T_a because the 11-year variation exists clearly in $\dot{H}$. The estimated T_a is 48 years. From Fig. 3, $T = 60$ years ($p = 0.92$) is obtained for $T_a/T_0 = 48/65 = 0.74$. Thus the period of the periodic term of $\dot{H}$ is approximately estimated at 60 years.

The spectrum $S_{1a}(p_1)$ of $\Delta\dot{H}_{1a}$ is calculated by (10) and (11) substituting observed values of $\Delta\dot{H}_{1a}$ for $\Delta y_{1a}(t)$. Black dots of Fig. 9 show the calculated $S_{1a}(p_1)$ which is

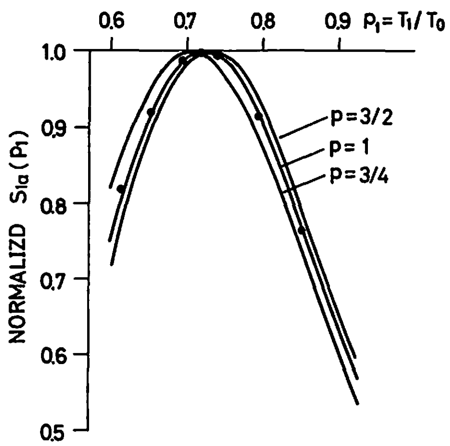

Fig. 9. Comparison between theoretical spectrum and observed one. Each spectrum is normalized as its maximum value is 1.

normalized as its maximum is 1. Curves of the figure are theoretically expected spectrums which also are normalized from those in Fig. 5. All the dots are near to the theoretical spectrum of $p = 1$ ($T=65$), but slightly shifted to $p < 1$. They are far from that of $p=3/4$ ($T\fallingdotseq 50$). Therefore best fit period T is estimated at about 60 years which is the same as that estimated from the apparent period T_a. Uncertainty of 5 years or so is inevitable in such period determination.

Substituting $T=60$ years ($p=0.92$) for period in (4),

$$\beta_1 - [\beta_1] = -0.0243a$$

is obtained. And then $\Delta\dot{H}_{1a}$ must be

$$\{-0.0243t + \sin(2\pi t/60)\}\, a$$

which gives

$$a = 16.4,\ \gamma/\mathrm{yr}$$

by the least square method from observed values of $\Delta\dot{H}_{1a}$. This a value and $[\beta_1] = 1.011\ \gamma/\mathrm{yr}^2$ determine the value of β_1,

$$\beta_1 = [\beta_1] - 0.0243a = 0.613\ \gamma/\mathrm{yr}^2.$$

Best fit period T can be determined also from symmetric term $\Delta\dot{H}_{1s}$ and values of coefficient β_0, β_2 and b may be obtained in similar way using this value of T, which may differ somehow from the above T value. However, the T value determined from $\Delta\dot{H}_{1a}$ is used here to determine these coefficients, because symmetric term is not suitable to deduce the period T as it is described in section 3. Thus determined values of the coefficients of symmetric term are,

$$b = 9.56\ \gamma/\mathrm{yr}$$
$$\beta_0 = -23.65\ \gamma/\mathrm{yr}$$
$$\beta_2 = 0.03248\ \gamma/\mathrm{yr}^3 .$$

All the coefficients give the best fit function of 10 observatories' mean secular variation,

$$\dot{H} = -23.7 + 0.613t + 0.0325t^2 + 16.4\sin(2\pi t/60) + 9.6\cos(2\pi t/60),\ \gamma/\mathrm{yr}, \quad (26)$$

where t is expressed in unit of year and $t = 0$ at 1932.5.

Direct method described in section 4 may be better to determine the period. Substituting observed values of $\dot{H}(t)$ for $y(t)$ in (22), $a_2(p_2)$ and $\beta_1(p_2)$ are calculated. Then the spectrum of anti-symmetric term, $S_{2a}(p_2)$, are obtained according to (24), where

$$\dot{H}_a(t) = \{\dot{H}(t) - \dot{H}(-t)\}/2$$

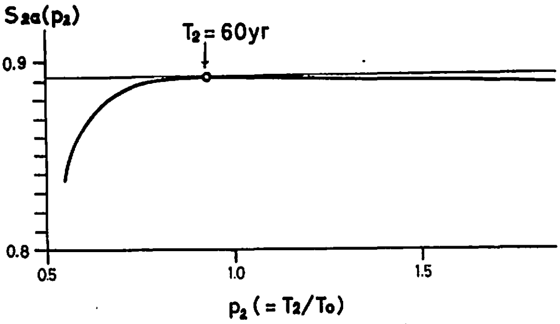

Fig. 10. Spectrum $S_{2a}(p_2)$ of $\dot{H}$.

is used for $y_a(t)$. Fig. 10 shows the calculated $S_{2a}(p_2)$ of $\dot{H}$, which has a very broad peak. Its mathematical maximum is found at $p_2 = 0.92$ which gives the best fit values,

$$T = p_2 T_0 = 60 \text{ years}$$
$$a = a_2(p_2) = 16.5\,\gamma/\text{yr}$$
$$\beta_1 = \beta_1(p_2) = 0.612\,\gamma/\text{yr}^2.$$

These values coincide with the above values determined by the other methods.

Three methods of determing the period give the same result for $\dot{H}$. Considering inevitable uncertainty in the determination, simpler way of T_a is convenient. Only this method is applied for $\dot{Z}$. First approximation of $\dot{Z}$,

$$\dot{Z}_1 = 14.32 + 0.0282t - 0.03191t^2 \tag{27}$$

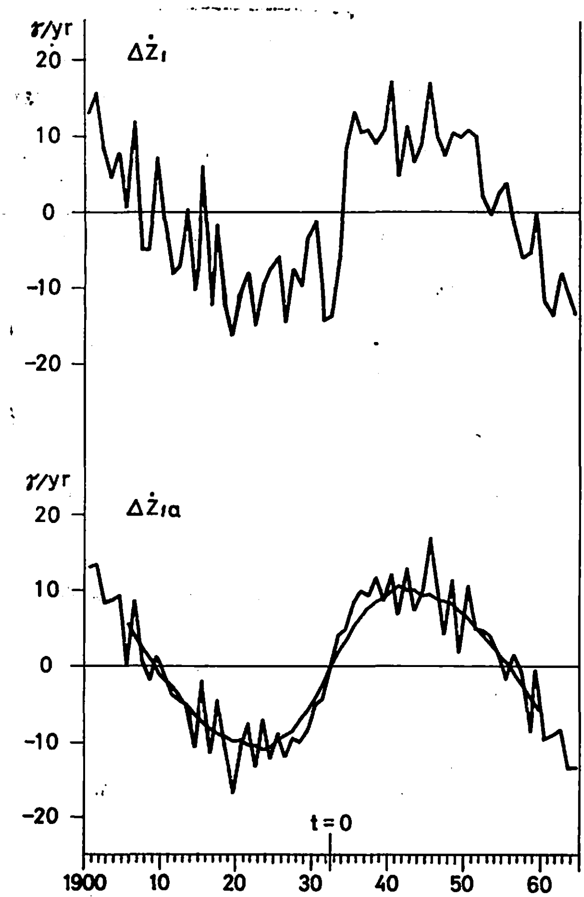

Fig. 11. Residual $\Delta\dot{Z}_1$ and its anti-symmetric term $\Delta\dot{Z}_{1a}$. 11-year running average also is shown for $\Delta\dot{Z}_{1a}$

is obtained from observed values shown in Fig. 7. Fig. 11 shows $\Delta\dot{Z}_1 = \dot{Z} - \dot{Z}_1$ and its anti-symmetric term $\Delta\dot{Z}_{1a}$. 11-year running average of $\Delta\dot{Z}_{1a}$ is shown too. Interval between zero-points of the 11-year running average gives $T_a = 47.5$ years, which is slightly smaller than that of $\Delta\dot{H}_{1a}$. This gives $T = 55$ years. Considering that the difference in T_a is only 0.5 years between $\dot{H}$ and $\dot{Z}$, that old data of $\dot{Z}$ is not so reliable and that the period determination includes some uncertainty, the period in $\dot{Z}$ is supposed here to be the same as that of $\dot{H}$; that is 60 years. Thus the best fit function of 10 observatories' mean secular variation for $\dot{Z}$ is expressed by,

$$\dot{Z} = 15.74 - 0.352t - 0.0274t^2 + 15.6\sin(2\pi t/60) - 2.05\cos(2\pi t/60),\ \gamma/\text{year}. \quad (28)$$

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地磁気永年変化における周期項と非周期項

柳 原 一 夫

概 要

地磁気永年変化に周期数十年の周期的変化の含まれることは既に指摘されている。観測された永年変化から非周期的変化を引き去れば，この周期的変化が明らかに認められるが，その周期、振幅はしばしば誤まり定められやすい。それは真の非周期的変化を直接求め難いからである。観測値を直接時間の二次式（あるいは一次式）で近似することを考え最小二乗法を適用したのでは，正しい非周期項はえられない。したがってこのときえられる非周期項を引いた残りの周期的変化ももちろん正しくない。これらのことはすべて観測期間と周期とが同程度であるためにおこる。

正しくは，二乗誤差を最小にするという原則を，周期項と非周期項の和という関数について，限定された期間の観測値に対して適用すべきである。この方法について考究し，現実の永年変化を解析した。中緯度に均等に分布する10カ所の観測所の平均永年変化から水平分力と垂直分力の年変化率の周期項として，周期60年，振幅 15～16 γ/yr がえられた。