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Base-Line Value Stability of Geomagnetic Variometer

By

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Abstract:—Base-line value changes of KM-type H-variometers used at three observatories, Kakioka, Memambetsu and Kanoya, for the 1960's are analysed. Their stabilities are good and the change in each variometer is expressed by a single function of time and temperature for a long time. In practical use of the relation, calculated values are as accurate as the absolute measurement by good magnetometers.

A new term of out-of-phase change is found in the relation for annual temperature variation. Though it is small in general, the term should be taken into consideration for precise determination of base-line values with accuracies better than 1γ.

1. Introduction

A typical system of geomagnetic routine observation consists of continuous variation measurement by the use of bar-magnet variometers and absolute measurement which determines the base-line value of the variometers. There's not much doubt about that the bar-magnet variometer will be used for a long time in future at many observatories, although modern electronic instruments are replacing old magnetic instruments. Because the variometer is simple in its mechanism, there is much merit in using it for observatory work. The cost of instrumentation and operation must be low. Therefore it is most desirable to obtain a good bar-magnet variometer as precise as modern electronic instruments without much increase of the cost.

A weak point of the present bar-magnet variometer is changes in its base-line value. Daily (or hourly) base-line values are determined by absolute measurements usually made once or twice a week in normal operation. Therefore the precision of continuous measurement of magnetic field depends upon the base-line value stability of the variometer, although the absolute measurement is much improved by the use of modern electronic instruments such as proton magnetometers.

Generally known factors of the base-line value change are temperature dependence and drift. In order to reduce the temperature dependence some kinds of compensation devices have been introduced into the design of variometers, and the room temperature is controlled with heat insulation materials or other devices. For the drift, which is generally a monotonic change with time, many improvements have been studied on materials of magnet and fiber, their annealing or seasoning and devices of suspension system.

By the improvements, the base-line value stability of recent variometers may be sufficient for the modern need of precise measurement. It is proved at least for a short-term stability in laboratory experiments. However the continuous operation in observatories needs long-term stability, because frequent changes or adjustments of variometer are not desirable. Meanwhile desired level of precision in observation becomes higher and higher with the progress in related fields of geophysics, and observers are burdened with the need of more precise determination of the base-line value for the variometer installed many years ago. Very old variometers sometimes show base-line value changes of a curious manner, particularly in the period in which the trend of annual temperature variation reverses from increase to decrease, suggesting an irreversible change or a hysteresis apart from a simple monotonic drift. Though modern variometers are believed to be free from this kind of large changes, small changes may still be giving obsevers difficulties in their attempt of obtaining an accuracy of better than 1 γ in base-line value determination.

A recent study of base-line value change made by Yamaguchi[1] shows that the change can be expressed by an empirical, analytical function of time and temperature with a sufficient accuracy for half a year or more including the season in which the trend in annual temperature variation changes. It is desirable to define the physical meaning of Yamaguchi's formulas and thereby making them valid at least for a year, *i.e.*, one cycle of annual temperature change.

The aim of the present paper is to study the long-term stability of base-line value of the variometers in routine use at our observatories, Kakioka, Memambetsu and Kanoya, and to help improve variometers and routine determination of base-line value. Our variometers, KM-type for horizontal (H) component and KZ-type for vertical (Z) component, were manufactured in the machine shop of Kakioka Magnetic Observatory in the 1960's on the basis of researches and experiments made in the 1950's. Since then they have been used for the routine observation. In this report base-line values of KM-type H-variometer are studied. Those of KZ-type Z-variometer will be reported in the next paper.

The KM-type H-variometer utilizes a bar-magnet of NKS-3 steel of Sumitomo Metal Co. which is equivalent to Alnico-5. Its diameter is 3 mm and the length is 10 mm. A small amount of temperature compensation alloy MS-2 (Ni 30, Cr 10 and Fe 60) of the same company is attached to the main magnet. By laboratory experiments before setting, the amount of MS-2 is adjusted so as to nullify the synthesized temperature coefficient of the variometer. Suspension fibre of the variometer is made of quartz of 60 μm in diameter. The upper end of the fibre is a small quartz bar which is tightly fixed to the torsion head with a chuck. The lower end is an inverse-T-shape quartz, which hangs an aluminium accessory for suspension of magnet and mirror.

Variometer houses of the three observatories are different from each other. Kakioka's variometer house consists of a granite inner house and a large shelter which covers the inner house with ample air space in between. A large amount of earth is piled up against the four walls of the shelter, burying the lower half which is made of concrete. Upper half of the shelter is a wooden structure with tiled roof. Heat insulation of Kakioka's variometer house is the best among the three. Its maximum annual range of the room temperature is about 8°C, from 12°C to 20°C. The annual variation is nearly sinusoidal without shorter-period fluctuations.

The variometer house of Memambetsu is a wooden house with double walls. Four layers of heat insulation material are attached to the inner side of the wall, ceiling and floor with air spaces between individual layers. Nevertheless, the maximum annual range of the room temperature is rather large (−4° to 19°C), and there remain fluctuations of a few degrees Celsius with a period of several days.

Kanoya's variometer house is made of concrete blocks, and half buried in the ground. Heat insulation material is attached to the inside walls of the house. The maximum annual range of the room temperature is about 21°C, from 8°C to 29°C*. Temperature fluctuations of a few degrees Celsius with a period of several days are also found.

Large changes in room temperature, such as those of Memambetsu and Kanoya, are not desirable for routine observation. However, adverse conditions

* In order to reduce large changes in room temperature, a new variometer house was constructed at Kanoya in 1973. Its basement floor is 6.3 m under the ground surface. Maximum annual range of the room temperature is less than a few degrees Celsius. A new variograph has been operating there since April 1973, though its data is not used in this report.

may be convenient for experimental studies of variometer. By this reason and because occasions of instrumental adjustments and earthquakes, which may cause discontinuous changes in base-line value, are less at Kanoya, its base-line values will be analysed here first. Then the method of analysis developed for Kanoya will be applied to the data of Kakioka and Memambetsu.

Two similar systems of variation measurement have been operated at each of the three observatories in order to prevent missed observations. One of the two systems is mainly used for routine data acquisition and the other is used at the time of instrumental trouble. Base-line values obtained by this system only are analysed in this report.

2. Base-Line Value of Kanoya's H-Variometer

A KM-type H-variometer was installed at Kanoya in December 1960. After three years of test operation, the variometer was put into routine use at the beginning of 1964. During the first 5 years and 9 months from January 1964 through September 1969, there were no discontinuous changes in base-line value and no instrumental adjustments. Data of this period are analysed here.

Fig. 1 shows monthly means of the observed base-line value and room temperature for the whole period. General trend of variation is seen from this figure, though daily values in any month fluctuate with shorter periods. All the years show nearly identical annual variation. Fig. 2 shows variation of the monthly

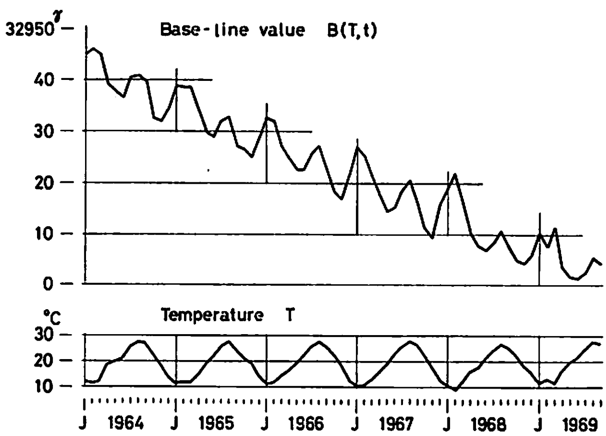

Fig. 1. Monthly means of observed base-line value $B(T, t)$ and room temperature T at Kanoya.

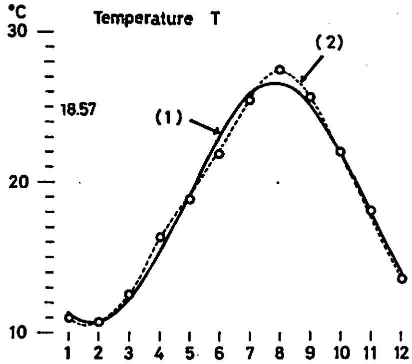

Fig. 2. Variation of monthly mean room temperature T at Kanoya. Approximate curve (1) is $18.57+7.95 \sin\{(2\pi/12)(t+8.2)\}$ and (2) is $(1)+0.74 \sin\{(2\pi/12)(2t+10.8)\}+0.48 \sin\{(2\pi/12)(3t+6.4)\}$, where $t=0$ for January. Abscissa is months of the year.

mean temperature (open circles) for 5 years from 1964 through 1968 together with the first term (full line) and the sum of the first, second and third terms (broken line) in its harmonic analysis. The first (annual) term is far larger than the others.

On the other hand the monthly mean base-line value of Fig. 1 shows a general trend of drift with a speed of about $-7\ \gamma$/year. Superposed upon the general

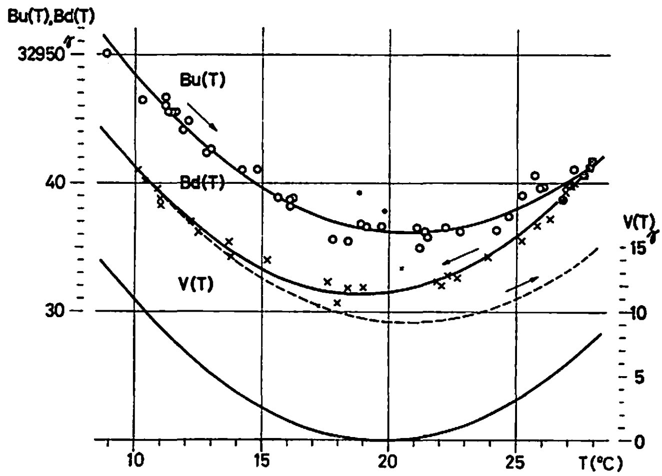

Fig. 3. Apparent temperature dependence of base-line value $B_u(T)$ for the months of ascending temperature and $B_d(T)$ for those of descending temperature and their mean $V(T)$ at Kanoya. Broken curve expresses $B_u(T)$ of the next year's which succeeds the $B_d(T)$, the former being not corrected for the year-to-year change.

drift, a semiannual variation is found. This may express temperature dependence of base-line value. Adding the corrections of the year-to-year drift, namely 7γ, 14γ, 21γ, 28γ and 35γ for the years 1965, 1966, 1967, 1968 and 1969, respectively, all the monthly mean base-line values are plotted in Fig. 3 against the room temperature. Circles and crosses in the figure express those in the months of ascending and descending temperature, respectively. Small circles and crosses represent the values in the months characterized by abnormal temperature change, such as April and May of 1964. These abnormal values are excluded from the following analysis.

Apparent temperature-dependent changes of base-line value, $B_u(T)$ for the period of ascending temperature and $B_d(T)$ for the period of descending temperature, are calculated by the least square method.

$$\left.\begin{aligned} B_u(T) &= \text{const} - 4.38T + 0.1055T^2 \quad (\text{in } \gamma) \\ B_d(T) &= \text{const} - 4.67T + 0.1235T^2 \quad (\text{in } \gamma) \end{aligned}\right\} \qquad (1)$$

Calculated $B_u(T)$ and $B_d(T)$ are shown in Fig. 3 by smooth curves. Broken line expresses $B_u(T)$ of the next year which lies just 7γ under the first $B_u(T)$, touching $B_d(T)$ at low temperatures. Both $B_u(T)$ and $B_d(T)$ include a drift component whose velocity is not necessarily constant throughout the year.

$B_d(T)$ nearly coincides with the next year's $B_u(T)$ for a rather long interval in the low temperature region. This means that there is no drift of base-line value in low temperatures. On the other hand, the gap between the two curves grows towards higher temperatures. The drift velocity of base-line value becomes larger in higher temperatures.

If the variation of room temperature T is symmetrical with respect to its maximum, the mean of $B_u(T)$ and $B_d(T)$ must represent the temperature dependence of base-line value irrespective of the temperature-dependent change of drift velocity. The symmetrical condition is nearly satisfied by the room temperature of Kanoya in monthly mean values as shown in Fig. 2. Therefore the temperature-dependent change of base-line value $V(T)$ is given by

$$\begin{aligned} V(T) &= \{B_u(T) + B_d(T)\}/2 \\ &= \text{const} - 4.53T + 0.1145T^2 \\ &= \text{const} - 0.273(T-T_0) + 0.1145(T-T_0)^2 \quad (\text{in } \gamma) \end{aligned} \qquad (2)$$

where $T_0 = 18.57\,^\circ\text{C}$ is the mean temperature. The temperature coefficient of base-line value,

$$\mathrm{d}V(T)/\mathrm{d}T = -0.273 + 0.2290(T-T_0) \quad (\text{in } \gamma/^\circ\text{C}) \qquad (3)$$

is depending upon temperature T. Bottom curve of Fig. 3 shows $V(T)$ with a

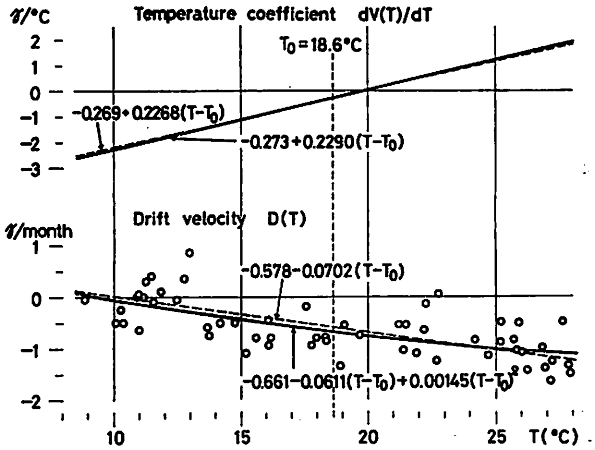

Fig. 4. Temperature coefficient $dV(T)/dT$ and drift velocity $D(T)$ at Kanoya. Broken lines are those obtained in Section 3.

constant so determined that the minimum of $V(T)$ should become zero. The temperature coefficient $dV(T)/dT$ is shown in the upper part of Fig. 4 by a straight full line. Broken line of the figure is the same coefficient calculated by using harmonic terms described in the next section.

Unfortunately the temperature coefficient changes with temperature. This may be a result of the use of a ready-made magnet. It can be much reduced by careful selection of material as those of Kakioka and Memambetsu. However

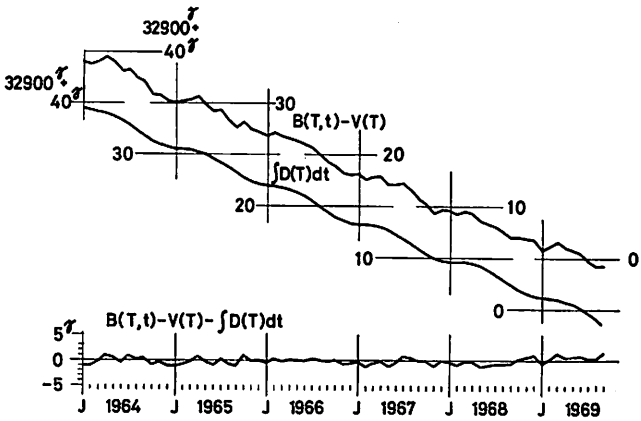

Fig. 5. $B(T,t)-\dot{V}(T)$, $\int D(T)dt$ and $B(T,t)-V(T)-\int D(T)dt$ at Kanoya.
$B(T,t)$: observed base-line value
$V(T)$: temperature dependence
$\int D(T)dt$: drift

the adverse condition of complicated temperature dependence gives a good opportunity for experimental study on base-line value stability.

Subtracting the known temperature-dependent part $V(T)$ from the observed value $B(T, t)$, the residual is the observed drift of base-line value, which is shown at the top of Fig. 5. The most conspicuous trend is a linear decrease. Superposed upon the linear decrease, a small but unmistakable annual variation is found. This means that the drift velocity shows an annual variation, generating an out-of-phase part of the complex temperature dependence of base-line value.

Monthly drifts are calculated from the difference between the values, $B(T, t) - V(T)$, one month before and one month after, and plotted in open circles at the bottom of Fig. 4 against the room temperature of the month. Scatter of each point of the figure is unavoidable because most of the monthly drift is smaller than 1γ. Drift velocity (monthly drift), $D(T)$, is given by

$$D(T) = -0.661 - 0.0611\,(T-T_0) + 0.00145\,(T-T_0)^2 \quad (\text{in } \gamma/\text{month}) \qquad (4)$$

As the last term of squared temperature contributes very little, the drift velocity almost linearly relates with temperature. The full line of the bottom figure in Fig. 4 shows $D(T)$ and the broken line shows the linear relation calculated in the next section by using harmonic analysis.

The drift component of base-line value is given by a time integral of $D(T)$, $\int_0^t D(T)\mathrm{d}t$, which is shown in Fig. 5. The sum of $V(T)$ and $\int_0^t D(T)\mathrm{d}t$ gives a calculated base-line value,

$$\begin{aligned} B_c(T, t) &= V(T) + \int_0^t D(T)\mathrm{d}t \\ &= \text{const} - 0.273(T-T_0) + 0.1145(T-T_0)^2 \\ &\quad + \int_0^t \{-0.661 - 0.0611(T-T_0) + 0.00145(T-T_0)^2\}\,\mathrm{d}t \end{aligned} \qquad (5)$$

Differences between the observed value $B(T, t)$ and the calculated value $B_c(T, t)$ are shown at the bottom of Fig. 5.

Standard deviation of the 69 difference values is 0.67γ for the whole period of 5 years and 9 months. This may include errors of slow fluctuations in the long period. Nevertheless the calculated values fit very well with the observed ones. The conditions for routine operation are not always good at Kanoya. The heat insulation of the variometer house is not sufficient and the temperature coefficient of the variometer is not constant for the actual temperature range. Even under such adverse conditions, the change in base-line value follows a single rule, as if there is no need of absolute measurement for a long time. In this sense, the variometer base-line value is stable.

Standard deviation of the difference between observed and calculated values becomes smaller for a shorter period. It is only 0.50γ for the middle two years, 1966 and 1967. This precision is as good as those of ordinary absolute instruments.

Foregoing results are based on monthly mean values. In order to check whether more rapid changes follow the same rule or not, individual observed base-line values of absolute measurement are compared with the calculated ones. In the year 1966, seventy-seven absolute measurements were made for base-line value determination. Distribution of the difference between observed and calculated base-line values is shown in Fig. 6. Its standard deviation is 0.64γ. All the difference values are within $\pm 1\gamma$ except a few extreme values. If two extreme values are excluded, the standard deviation is reduced to 0.56γ. Errors are so small that Eq. (5) is applicable.

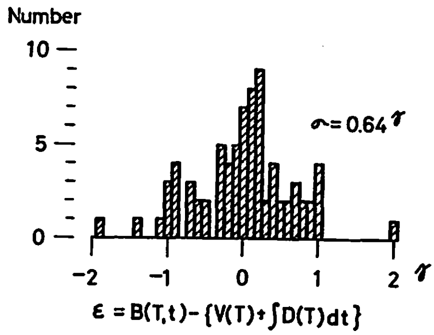

Fig. 6. Distribution of the difference between observed values $B(T, t)$ and calculated ones $V(T)+\int D(T)dt$ of the base-line value at Kanoya for the year 1966.

Each one of the 77 observed base-line values is the mean of 4 measured values in a day. From the distribution of the 4 values, the random error of one observed base-line value is estimated at 0.47γ, which is the square root of mean variance. This random error must be included in the above-mentioned error of 0.56γ. Therefore a much smaller error should be ascribed to the calculation by Eq. (5).

Rapid changes of base-line value are examined again for short-period fluctuations of room temperature. For a single month or two, base-line values may be expressed by a linear function of temperature T and time t,

$$B(T, t) = \text{const} + \xi T + \eta t \qquad (6)$$

When T changes randomly with respect to t, the temperature coefficient ξ can be estimated with a sufficient accuracy. Good heat insulation of the variometer house becomes a disadvantage for this purpose. The drift velocity η is generally so small that its accurate value is not obtained from data of a short period. For the year 1964, seven periods of adequate temperature change are selected. Each temperature coefficient ξ of the periods is calculated by the least square method and shown in Fig. 7 for the mean temperature of the respective periods. Mean

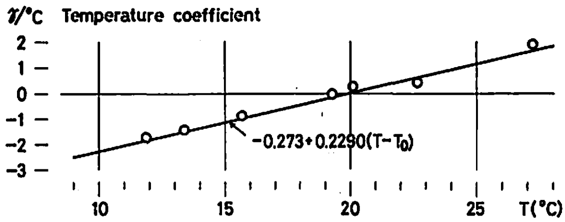

Fig. 7. Temperature coefficients calculated by Eq. (6) for about a month (circles). They fall closely on the straight line given by Eq. (3).

length of the periods is about one month, and the number of absolute measurements in one period is from 5 to 7. Mean range of temperature is 2.7°C. As it is shown in Fig. 7, present values of temperature coefficient coincide very well with the straight line given by Eq. (3) which is obtained from monthly mean values. Therefore it is again proved that there is no difference between temperature coefficients of slow and rapid changes.

In the year 1964 when the new variometer was put into routine use, it must have been impossible to analyse the base-line value by the same way as is descrived in the forepart of this section. An alternative is the method described just above. Generally base-line values of a new variometer will be analysed in the same way and daily values can be determined by using the known temperature coefficient. If temperature changes are proportional to time t, the coefficient ξ will not be separated from η. There is no need of the separation in this case for the purpose of practical base-line value determination in routine operation.

3. Harmonic Analysis of Temperature and Base-Line Value

Harmonic analysis may be a convenient way to obtain the relation between the base-line value and room temperature because the latter changes periodically with a fundamental period of one year, t_0, given by

$$t_0 = 2\pi/p \qquad (7)$$

The variation of room temperature is expressed by

$$T = T_0 + \sum_{n=1}^{\infty} T_n \sin(npt + \varphi_n) \tag{8}$$

On the other hand, the variation of base-line value includes a non-cyclic term of drift βt, and is given by

$$\begin{aligned} B &= \beta t + B_0 + \sum_{n=1}^{\infty} B_n \sin(npt + \psi_n) \\ &= \beta t + B_0 + \sum_{n=1}^{\infty} B_{ns} \sin(npt + \varphi_n) + \sum_{n=1}^{\infty} B_{nc} \cos(npt + \varphi_n) \end{aligned} \tag{9}$$

where

$$\left.\begin{aligned} B_{ns} &= B_n \cos(\psi_n - \varphi_n) \\ B_{nc} &= B_n \sin(\psi_n - \varphi_n) \end{aligned}\right\} \tag{10}$$

If $B_{nc}=0$ and $B_{ns}/T_n=$const, both of the temperature coefficient and the drift velocity of the variometer are constant irrespective of temperature. This is not so in many actual variometers. As a generalization, it is supposed here that both of temperature coefficient and drift velocity linearly relate to temperature. The variation of base-line value in this generalized case is given by

$$B(T, t) = \text{const} + b(T-T_0) + c(T-T_0)^2 + \int_0^t \{h + k(T-T_0)\}\, dt \tag{11}$$

This expression is very similar to Kanoya's experimental case of Eq. (5), whose last term of the time integral is negligible, where b, c, h and k are constants. Substituting Eq. (8) for T in Eq. (11),

$$\begin{aligned} B(T, t) = \text{const} &+ ht + b \sum_{n=1}^{\infty} T_n \sin(npt + \varphi_n) - \frac{k}{np} \sum_{n=1}^{\infty} T_n \cos(npt + \varphi_n) \\ &- \frac{c}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} T_n T_m \sin(\varphi_n - \varphi_m - \varphi_{n-m}) \sin\{(n-m)pt + \varphi_{n-m}\} \\ &+ \frac{c}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} T_n T_m \cos(\varphi_n - \varphi_m - \varphi_{n-m}) \cos\{(n-m)pt + \varphi_{n-m}\} \\ &+ \frac{c}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} T_n T_m \sin(\varphi_n + \varphi_m - \varphi_{n+m}) \sin\{(n+m)pt + \varphi_{n+m}\} \\ &- \frac{c}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} T_n T_m \cos(\varphi_n + \varphi_m - \varphi_{n+m}) \cos\{(n+m)pt + \varphi_{n+m}\} \end{aligned} \tag{12}$$

is obtained, where

$$\varphi_{i-i} = \varphi_0 = 0, \qquad \varphi_{i-j} = -\varphi_{j-i} \quad (j > i) \tag{13}$$

As far as monthly mean values are concerned, higher harmonic terms of temperature T are much smaller than the fundamental term T_1,

$$T_1 \gg T_2, T_3 \ldots\ldots\ldots \tag{14}$$

Ignoring higher order small quantities, $(T_i/T_1)\,(T_j/T_1)$ $(i \geqq 2,\ j \geqq 2)$, Eq. (12) is reduced to

$$
\begin{aligned}
B(T,t) = &\text{const} + ht \\
&+\{b+cT_2\sin(2\varphi_1-\varphi_2)\}\, T_1 \sin(pt+\varphi_1) \\
&+\left\{cT_2\cos(2\varphi_1-\varphi_2)-\frac{k}{p}\right\} T_1\cos(pt+\varphi_1) \\
&+\left[bT_2+cT_1\left\{\frac{T_1}{2}\sin(2\varphi_1-\varphi_2)-T_3\sin(\varphi_3-\varphi_1-\varphi_2)\right\}\right]\sin(2pt+\varphi_2) \\
&+\left[cT_1\left\{-\frac{T_1}{2}\cos(2\varphi_1-\varphi_2)+T_3\cos(\varphi_3-\varphi_1-\varphi_2)\right\}-\frac{k}{p}\frac{T_2}{2}\right]\cos(2pt+\varphi_2) \\
&+\left[bT_3+cT_1\left\{T_2\sin(\varphi_1+\varphi_2-\varphi_3)-T_4\sin(\varphi_4-\varphi_1-\varphi_3)\right\}\right]\sin(3pt+\varphi_3) \\
&+\left[cT_1\{-T_2\cos(\varphi_1+\varphi_2-\varphi_3)+T_4\cos(\varphi_4-\varphi_1-\varphi_2)\}-\frac{k}{p}\frac{T_3}{3}\right]\cos(3pt+\varphi_3) \\
&+\cdots\cdots \qquad (15)
\end{aligned}
$$

Each term of Eq. (15) should be compared with the respective term of Eq. (9). The following equations,

$$
\left.
\begin{aligned}
B_1\cos(\psi_1-\varphi_1) &= B_{1s}=\{b+cT_2\sin(2\varphi_1-\varphi_2)\}\,T_1 \\
B_1\sin(\psi_1-\varphi_1) &= B_{1c}=\left\{cT_2\cos(2\varphi_1-\varphi_2)-\frac{k}{p}\right\}T_1 \\
B_2\cos(\psi_2-\varphi_2) &= B_{2s}= bT_2+cT_1\left\{\frac{T_1}{2}\sin(2\varphi_1-\varphi_2)-T_3\sin(\varphi_3-\varphi_1-\varphi_2)\right\} \\
B_2\sin(\psi_2-\varphi_2) &= B_{2c}= cT_1\left\{-\frac{T_1}{2}\cos(2\varphi_1-\varphi_2)+T_3\cos(\varphi_3-\varphi_1-\varphi_2)\right\}-\frac{k}{p}\frac{T_2}{2} \\
B_3\cos(\psi_3-\varphi_3) &= B_{3s}= bT_3+cT_1\{T_2\sin(\varphi_1+\varphi_2-\varphi_3)-T_4\sin(\varphi_4-\varphi_1-\varphi_3)\} \\
B_3\sin(\psi_3-\varphi_3) &= B_{3c}= cT_1\{-T_2\cos(\varphi_1+\varphi_2-\varphi_3)+T_4\cos(\varphi_4-\varphi_1-\varphi_3)\}-\frac{k}{p}\frac{T_3}{3}
\end{aligned}
\right\} \cdots\cdots (16)
$$

and

$$\beta = h \qquad (17)$$

are obtained.

Three unknown factors b, c and k of the equations in (16) can be calculated by the least square method applied for significant B_n terms. Even if all the higher harmonics, T_2, T_3......, of T are ignored, the second harmonic term appears in B, coming from the temperature dependence c of the temperature coefficient. And the out-of-phase term, B_{1c}, of the fundamental period appears also due to the temperature dependence k of the drift velocity. Therefore at least first four equations in (16) should be used to obtain the values of b, c and k.

Present method of analysis is first applied for Kanoya's data used in the

Table 1. Harmonic analysis of monthly mean values of temperature and base-line value of H-variometer at Kanoya, 1964–1968.

	Temperature			Base-line value				
	T_n	φ_n		B_n	B_{ns} obs.	B_{ns} cal.	B_{nc} obs.	B_{nc} cal.
	℃	°	month	γ	γ	γ	γ	γ
$n=1$	7.95	244.6	(8.2)	2.04	−2.00	−1.98	0.42	0.42
2	0.74	323.2	(10.8)	4.00	0.90	0.79	3.90	3.94
3	0.48	192.1	(6.4)	0.63	0.43	0.23	−0.47	−0.52
4	0.23	15.0	(0.5)	0.19				

Mean temperature $T_0=18.57$℃, non-cyclic change $\beta=-0.578\,\gamma$/month, fundamental period $t_0=12$ months ($t=0$ at January).

Values of B_{ns} and B_{nc} shown in the "cal." column are those calculated by the equations in (16) with coefficients b, c and k given in (18).

previous section. Table 1 shows results on harmonic analysis for monthly mean values of the room temperature and the base-line value during 5 years from 1964 to 1968. In the harmonics of temperature T, T_1 alone is large and all the other harmonics are small. The 4th and higher terms are negligible. On the other hand, the 2nd harmonic component of the base-line value is larger than the 1st one. This means the temperature coefficient of the variometer depends upon temperature as has been mentioned above. Because B_3 is not negligible, first 6 equations in (16) are used to calculate the values of coefficients b, c and k by the least square method. Calculated values are

$$b=-0.269\,\gamma/°\mathrm{C},\quad c=0.1134\,\gamma/(°\mathrm{C})^2,\quad k=-0.0702\,\gamma/\mathrm{month}/°\mathrm{C} \tag{18}$$

Values of B_{ns} and B_{nc} calculated by the use of these values are shown in the "cal." column of Table 1.

The temperature coefficient and drift velocity, both temperature-dependent, are expressed by

$$\mathrm{d}V(T)/\mathrm{d}T=-0.269+0.2268(T-T_0)\qquad(\gamma/°\mathrm{C}) \tag{19}$$

and

$$D(T)=-0.578-0.0702(T-T_0)\qquad(\gamma/\mathrm{month}) \tag{20}$$

respectively, and shown in Fig. 4 by broken line. They coincide very well with the results of the previous section as a matter of course.

4. Base-Line Value of Kakioka's H-Variometer

A KM-type H-variometer was installed at Kakioka in June 1962. Unfortunately, there are many earthquakes whose mechanical shocks are apt to cause discontinuous changes in the variometer base-line value at Kakioka. It is difficult to find substantial length of periods free from any discontinuous change, which will be called "gap" of base-line value hereafter. During two years from March 1968 to March 1970, there was no large gap except small ones. Base-line values of the two years are analysed here. The magnitude of the small gap is estimated by using the jump in magnetogram trace, the difference between observed values of absolute measurements before and after and comparison with the other magnetogram of the same day. Observed base-line values are corrected by the magnitude of the gap so as to make discontinuities disappear. The number of small gaps is 21 against 124 felt earthquakes, and the total of the magnitude of the gaps over the 2 years is $+5.2\gamma$. Corrected base-line values are shown in Fig. 8, where each γ value has been rounded to unit digit because the correction of gaps may have introduced some ambiguity in the fraction of γ.

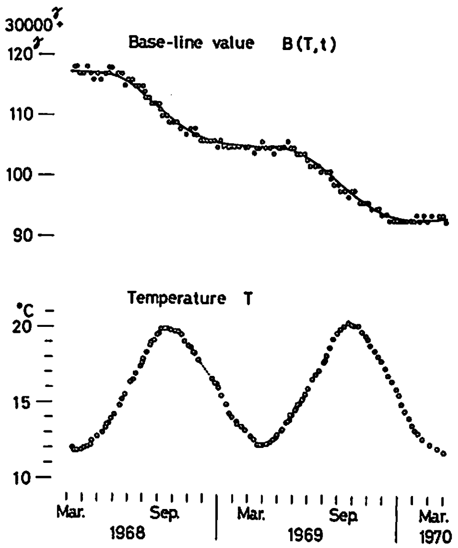

Fig. 8. Observed base-line value $B(T, t)$ (corrected by the magnitude of gap) and room temperature T at Kakioka.

As the heat insulation of the variometer house at Kakioka is good, the variation of the room temperature is fairly smooth and approximately sinusoidal (Fig. 8). Amplitude of the variation is small and no short-period fluctuation is found.

Table 2 shows results on harmonic analysis of monthly mean values of the room temperature and the base-line value. Only the fundamental term is con-

Table 2. Harmonic analysis of monthly mean values of temperature and base-line value of H-variometer at Kakioka, March 1968–March 1970.

	Temperature			Base-line value				
	T_n	φ_n		B_n	B_{ns} obs.	B_{ns} cal.	B_{nc} obs.	B_{nc} cal.
	℃	°	month	γ	γ	γ	γ	γ
$n=1$	3.87	198.5	(6.6)	2.11	−0.87	−0.87	1.92	1.92
2	0.16	33.2	(1.1)	0.13	−0.06	0.00	−0.11	0.02
3	0.14	225.2	(7.5)	0.19				

Mean temperature $T_0=16.07$℃, non-cyclic change $\beta=-0.988\,\gamma$/month, fundamental period $t_0=12$ months ($t=0$ at January).
Values of B_{ns} and B_{nc} shown in the "cal." column are those calculated by the equations in (16) with coefficients b, c and k given in (21).

spicuous for both temperature and base-line value. Higher harmonic terms are nearly negligible. The in-phase term B_{1s} of base-line value is smaller than 1γ. This means that the temperature coefficient of base-line value is nearly negligible. On the other hand, the out-of-phase term B_{1c} is larger than the in-phase term B_{1s}. Therefore the drift velocity must change with temperature.

Taking the first 4 equations in (16), calculations of the least square method give the values of coefficients b, c and k as follows.

$$b=-0.227\,\gamma/°\mathrm{C}, \quad c=0.0053\,\gamma/(°\mathrm{C})^2, \quad k=-0.2592\,\gamma/\mathrm{month}/°\mathrm{C}. \tag{21}$$

Values of B_{ns} and B_{nc} calculated by the use of these values of the coefficients are shown in the "cal." column of Table 2. The temperature coefficient of the variometer,

$$\mathrm{d}V(T)/\mathrm{d}T=-0.227+0.0106(T-T_0) \quad (\text{in } \gamma/°\mathrm{C}) \tag{22}$$

is very small and nearly constant for the full range of the room temperature. On the other hand, the drift velocity,

$$D(T)=-0.988-0.2592(T-T_0) \quad (\text{in } \gamma/\mathrm{month}) \tag{23}$$

is larger than that of Kanoya, and depends upon temperature.

Smooth curve of Fig. 8 expresses base-line values calculated by Eq. (11) with the coefficients given in (21). Standard deviation of the difference between observed and calculated monthly mean base-line values is only 0.45γ for the entire two years. In spite of existence of many small gaps, the variation of base-line value is simple and systematic if it is corrected by the magnitude of the gap.

5. Base-Line Value of Memambetsu's H-Variometer

A KM-type H-variometer was installed at Memambetsu in October 1962. Occasions of discontinuous change (gap) of the base-line value are not so many at Memambetsu as at Kakioka, but more than those at Kanoya. During two years from February 1965 to February 1967 there was no such gap at Memambetsu. Monthly mean values of the two years are analysed here. Although each observed base-line value changes in a month according to rather large fluctuations of the room temperature, the temperature coefficient must be the same both for slow and rapid changes as it is known from the analysis of Kanoya's KM-variometer (Section 2). Fig. 9 shows monthly mean values of the base-line value and the room temperature. Results on harmonic analysis of the monthly mean values are given in Table 3.

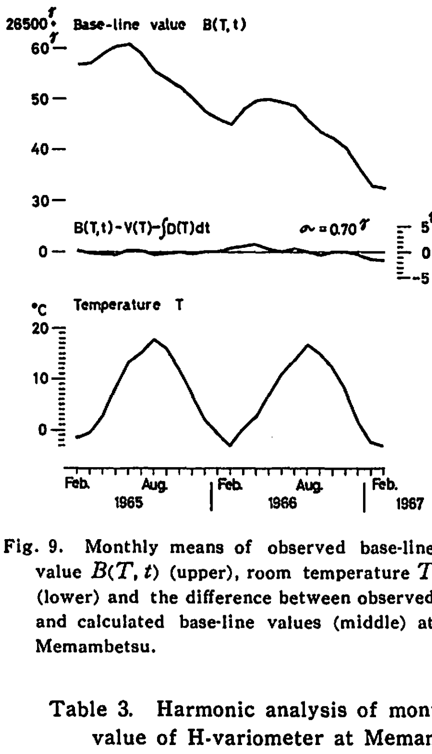

Fig. 9. Monthly means of observed base-line value $B(T, t)$ (upper), room temperature T (lower) and the difference between observed and calculated base-line values (middle) at Memambetsu.

Only the fundamental term T_1 is large for temperature. On the other hand, the 2nd harmonic term also is

Table 3. Harmonic analysis of monthly mean values of temperature and base-line value of H-variometer at Memambetsu, February 1965–February 1967.

	Temperature			Base-line value				
	T_n	φ_n		B_n	B_{ns} obs.	B_{ns} cal.	B_{nc} obs.	B_{nc} cal.
	℃	°	month	γ	γ	γ	γ	γ
$n=1$	9.49	240.6	(8.0)	3.83	2.96	2.95	2.43	2.43
2	0.49	326.2	(10.9)	1.13	0.16	0.27	−1.12	−1.11
3	0.10	15.0	(0.5)	0.39				

Mean temperature $T_0=7.28$℃, non-cyclic change $\beta=-0.885\,\gamma$/month, fundamental period $t_0=12$ months ($t=0$ at January).
Values of B_{ns} and B_{nc} shown in the "cal." column are those calculated by the equations in (16) with coefficients b, c and k given in (24).

significant for base-line value. Taking the first four equations in (16), values of coefficients b, c and k are calculated by the least square method. The calculated values are

$$b = 0.310\,\gamma/^{\circ}\mathrm{C}, \quad c = -0.0257\,\gamma/(^{\circ}\mathrm{C})^2, \quad k = -0.1274\,\gamma/\mathrm{month}/^{\circ}\mathrm{C}. \qquad (24)$$

Values of B_{ns} and B_{nc} calculated by the use of these values of the coefficients are given in the "cal." column of Table 3.

The temperature coefficient of the base-line value,

$$\mathrm{d}V(T)/\mathrm{d}T = 0.310-0.0514(T-T_o) \qquad (\text{in } \gamma/^{\circ}\mathrm{C}) \qquad (25)$$

and the drift velocity

$$D(T) = -0.885-0.1274(T-T_o) \qquad (\text{in } \gamma/\mathrm{month}) \qquad (26)$$

are median between those of Kakioka and Kanoya.

Differences between observed and calculated base-line values are shown in the middle of Fig. 9. A rather systematic variation of the difference is found in the last half of the two-year period. It may be an additional drift which is not expressed by Eq. (11). Including this abnormal drift, the standard deviation of the difference is 0.70γ for the whole period. This value is not so bad. For the first one year, the standard deviation of the difference is only 0.35γ. In the practical use of the analysis for base-line value determination, such abnormal drift is known from the residual curve and can be extracted, if it is smooth and systematic.

6. Discussion and Concluding Remark

Generally changes in base-line value can be expressed by Eq. (11) with a sufficient accuracy. Temperature coefficients of KM-type H-variometers are all small with shunt alloy used for temperature compensation, but some of them change with temperature. A new term of temperature dependence of drift velocity is introduced into the empirical expression of base-line values.

It is desirable to have experiments on material characteristics in laboratory before setting the variometer and to compare them with those of the variometer in routine operation. Unfortunately, laboratory experiments were not enough in the present case for accounting for the variometer characteristics, especially on the temperature dependence of drift. The value of temperature coefficient seems to be a reasonable one judging from the known material characteristics and the practical adjustment of the amount of shunt alloy. On the other hand, physical origin of the drift is less known.

The mean drift is a monotonic decrease of base-line value. Similar negative drifts are found also in many variometers other than KM-type Eschenhagen variometers made by Askania Werke, for example. The negative drift seems to be independent of the use of shunt alloy for temperature compensation. From studies on base-line value for many variometers of different kinds, Kuboki[(2)] concluded that the base-line value of H-variometer decreases exponentially during about one year from the initial installation and then it generally reaches a constant negative drift with a velocity of several or a few γ per year.

If the moment of magnet decreases, the deflection angle of the H-variometer must increase and the light spot should move in the direction of H decrease on the recorder. This is nothing other than the increase of base-line value, which is the reverse of the observed drift. Long-term, continuous increase of the moment, which can account for the observed drift, is difficult to be supposed as a matter of course. Secular changes in torsion constant of quartz fibre are generally too small to explain the observed drift.

Old variometers have often shown large and unstable drifts which sometimes are due to insufficient fixing of fibre at the torsion head or at the magnet. For KM-variometers, the fixing is carefully designed not to allow relaxation. As a result the drift of the variometer becomes small and systematic as described in the preceding sections. However it is not zero yet.

Temperature dependence of the drift is more difficult to explain. It has been overlooked before because the double amplitudes of the drift velocity variation,

$$-2kT_1 = 2.4\ \gamma/\text{month for Memambetsu}$$
$$2.0 \quad \text{for Kakioka}$$
$$1.1 \quad \text{for Kanoya}$$

are all small. But the important point is that it represents the out-of-phase part of the temperature dependence of the variometer. The response of base-line value includes an out-of-phase part as well as an in-phase part for temperature change. Sometimes the former is comparable to the latter, particularly when the in-phase part (temperature coefficient) becomes small as that of Kakioka. In order to study the physical origin of the out-of-phase part, more strict response analyses in complex representation are necessary. In the present simple analysis, k is assumed to be a constant for any frequency of temperature change, though it might change with frequency. This simplification might cause the differences between observed and calculated B_{ns} or B_{nc} for $n=2$ or 3, which are larger than those for $n=1$, in Tables 1, 2 and 3. Further studies need laboratory experiments because the routine data are insufficient for the purpose of fine response study.

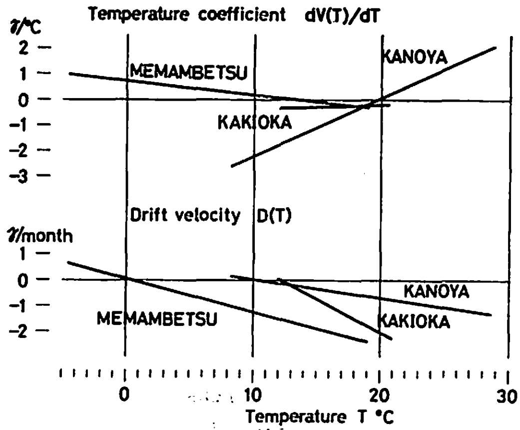

Fig. 10. Summary of temperature coefficients $dV(T)/dT$ and drift velocities $D(T)$ at three observatories.

Temperature coefficients $dV(T)/dT$ and drift velocities $D(T)$ of the three variometers are shown in Fig. 10. Each end of the straight lines expresses the maximum or the minimum of the room temperature in the period of the analysed data. Most of the room temperatures fall between T_o-T_1 and T_o+T_1. The mean of the absolute value of the temperature coefficient for this temperature range,

$$\overline{|dV(T)/dT|}=\frac{1}{2T_1}\int_{T_0-T_1}^{T_0+T_1}|dV(T)/dT|dT=0.23\,\gamma/°\mathrm{C}\quad \text{for Kakioka}$$

$$0.34\quad \text{for Memambetsu}$$

$$0.92\quad \text{for Kanoya,}$$

is a practical measure of the temperature dependence (in-phase part) in routine

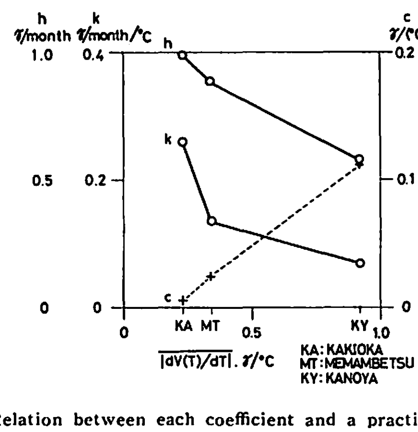

Fig. 11. Relation between each coefficient and a practical measure of temperature dependence, $\overline{|dV(T)/dT|}$.

operation. The temperature coefficient of the temperature coefficient c is nearly proportional to $\overline{|dV(T)/dT|}$ as is shown in Fig. 11. On the other hand, both of the mean drift velocity $h(=\beta)$ and the temperature coefficient of the drift velocity k are in the inverse relation with $\overline{|dV(T)/dT|}$. Present analysis of the three variometers suggests that the better the temperature dependence (in-phase part) is, the larger the drift velocity becomes, and *vice versa*, although three cases are not enough to draw a general conclusion from.

Eq. (11) is useful for routine determination of base-line value. The coefficients, b, c, h and k, of a good variometer are constant for a long time and these values are obtained from the analysis described in Sections 2 and 3. Calculated values are checked by weekly absolute measurements. Any out-of-rule change indicates bad condition of the variometer. Determination of the coefficient value may be made for running periods to watch any gradual change in the value.

Considering that temperature coefficients (in-phase part) may be more stable than drifts, it is a convenient way to subtract the calculated value of $b(T-T_0)+c(T-T_0)^2$ from the observed value of absolute measurement. Smoothing the remainder, the drift of base-line value is obtained. This is the same as ordinary procedure of base-line value determination[3]. However the existence of the out-of-phase part of temperature dependence should be taken into consideration in calculating the coefficients b and c and obtaining smooth curve of the drift.

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地磁気変化計の安定性

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われわれの3観測所，柿岡，女満別，鹿屋の水平分力変化観測には1960年代からKM型変化計を用いてきた．この変化計の基線値を解析した結果，各変化計の基線値変化は長い間にわたって時間と温度に関する一定の関数であらわせることがわかった．この関数を使って計算した基線値は変化計がよい状態にある限り絶対観測による測定値と同程度もしくはそれ以上の精度をもつと考えられる．

関数関係の中には従来から知られているもののほかに温度年変化に対して位相がずれて対応する変化分があることに留意しなければならない．精度の高い基線値決定の際には考慮すべきことであろう．

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